Chapter 6: Quadratic Equations

Getting Started, page 310

1. The vertex is (–1, 8), the axis of symmetry is $x = -1$, and the zeros are –3 and 1.

2. a) The equation must be of the form $y = ax^2 + bx + c$. The equation in this form is iii).

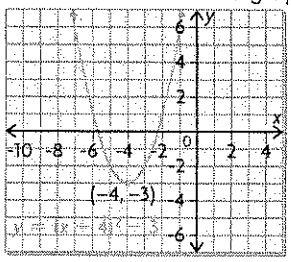
b) The equation must be of the form $y = a(x - r)(x - s)$. The equation in this form is i).

c) The equation must be of the form $y = a(x - h)^2 + k$. The equation in this form is ii).

3. a) The equation is in vertex form.
Vertex: (–4, –3)
Axis of symmetry: $x = -4$
Let x equal 0 to determine the y-intercept:

$$y = (x + 4)^2 - 3$$
$$y = (0 + 4)^2 - 3$$
$$y = 16 - 3$$
$$y = 13$$

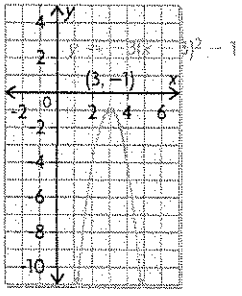
Use this information to graph the quadratic relation:



b) The equation is in vertex form.
Vertex: (3, –1)
Axis of symmetry: $x = 3$
Let x equal 0 to determine the y-intercept:

$$y = -3(x - 3)^2 - 1$$
$$y = -3(0 - 3)^2 - 1$$
$$y = -3(9) - 1$$
$$y = -28$$

Use this information to graph the quadratic relation:



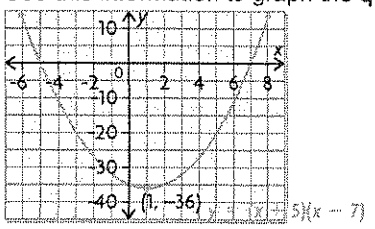
c) The equation is in factored form.
Zeros: $x = -5$, $x = 7$

Axis of symmetry: $x = \dfrac{-5 + 7}{2} = 1$

Substitute the x-coordinate of the vertex, $x = 1$, into the equation to determine the y-coordinate of the vertex:

$$y = (x + 5)(x - 7)$$
$$y = (1 + 5)(1 - 7)$$
$$y = (6)(-6)$$
$$y = -36$$

Use this information to graph the quadratic relation:



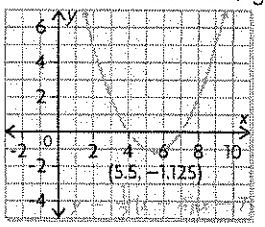
d) The equation is in factored form.
Zeros: $x = 4$, $x = 7$

Axis of symmetry: $x = \dfrac{4 + 7}{2} = 5.5$

Substitute the x-coordinate of the vertex, $x = 5.5$, into the equation to determine the y-coordinate of the vertex:

$$y = \frac{1}{2}(x - 4)(x - 7)$$
$$y = \frac{1}{2}(5.5 - 4)(5.5 - 7)$$
$$y = \frac{1}{2}(1.5)(-1.5)$$
$$y = -1.125$$

Use this information to graph the quadratic relation:



e) The equation is in standard form.
y-intercept: –1
Use partial factoring to determine two ordered pairs with the same y-coordinate. Then, determine the axis of symmetry by calculating the mean of the x-coordinates:

$$y = 2x^2 + x - 1$$
$$y = x(2x + 1) - 1$$

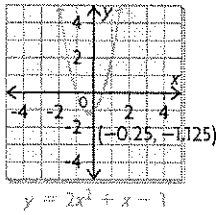
$x = 0$ and $x = -0.5$
Two ordered pairs are (0, –1) and (–0.5, –1).

The axis of symmetry is $x = \dfrac{-0.5 + 0}{2} = -0.25$.

Substitute this value into the equation to determine the y-coordinate of the vertex.

$$y = 2(-0.25)^2 + (-0.25) - 1$$
$$y = 0.125 - 0.25 - 1$$
$$y = -1.125$$

Use this information to graph the quadratic relation:



f) The equation is in standard form.
y-intercept: 0
Use partial factoring to determine two ordered pairs with the same y-coordinate. Then, determine the axis of symmetry by calculating the mean of the x-coordinates:

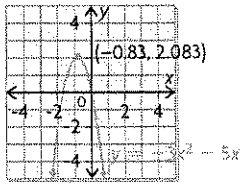
$$y = -3x^2 - 5x$$
$$y = x(-3x - 5)$$

$x = 0$ and $x = -\frac{5}{3}$

Two ordered pairs are $(0, 0)$ and $\left(-\frac{5}{3}, 0\right)$.

The axis of symmetry is $x = \frac{-\frac{5}{3} + 0}{2} \doteq -0.83$.

Substitute this value to determine the y-coordinate of the vertex:
$y = -3(-0.83)^2 - 5(-0.83)$
$y = -2.07 + 4.15$
$y = 2.08$
Use this information to graph the quadratic relation:



4. a) This is a trinomial where $a = 1$. To determine its factors, determine two numbers whose product is 12 and whose sum is 8. The numbers are 6 and 2, so the factored expression is $(x + 6)(x + 2)$.
b) This is a trinomial where $a = 1$. To determine its factors, determine two numbers whose product is 6 and whose sum is –5. The numbers are –2 and –3, so the factored expression is $(x - 2)(x - 3)$.
c) This is a trinomial where $a = 1$. To determine its factors, determine two numbers whose product is –30 and whose sum is 7. The numbers are 10 and –3, so the factored expression is $(x + 10)(x - 3)$.
d) This trinomial is a perfect square because it is of the form $a^2 - 2ab + b^2$. The root of the first term is $3x$, the root of the last term is 5 and the sign on the second term is negative, therefore the factored expression is $(3x - 5)^2$.
e) This is a trinomial where $a \neq 1$. Determine two numbers whose product is $(-6)(24) = -144$ and whose sum is –7. The numbers are 9 and –16. Use these to decompose the middle term and then factor:

$$\begin{aligned} -6x^2 - 7x + 24 &= -6x^2 + 9x - 16x + 24 \\ &= 3x(-2x + 3) + 8(-2x + 3) \\ &= (3x + 8)(-2x + 3) \\ &= -(3x + 8)(2x - 3) \end{aligned}$$

f) This is a trinomial where $a \neq 1$. To determine its factors, determine two numbers whose product is $2(-5) = -10$ and whose sum is –1. There are no such numbers, so the expression cannot be factored over the integers.

5. a) $4x + 8 = 0$
$4x = -8$
$x = -2$

b) $5x - 3 = 0$
$5x = 3$
$x = \frac{3}{5}$

c) $-2x + 12 = 0$
$-2x = -12$
$x = 6$

d) $12x + 7 = 0$
$12x = -7$
$x = -\frac{7}{12}$

6. a) $(3x - 5)(x - 4) = 3x^2 - 12x - 5x + 20$
$= 3x^2 - 17x + 20$

b) $(n + 1)(n - 1) = n^2 + n - n - 1$
$= n^2 - 1$

c) $(2x + 3)(4x - 5) = 8x^2 - 10x + 12x - 15$
$= 8x^2 + 2x - 15$

d) $(7 - 3p)(2p + 5) = 14p + 35 - 6p^2 - 15p$
$= -6p^2 - p + 35$

e) $(3a + 7)(3a + 7) = 9a^2 + 21a + 21a + 49$
$= 9a^2 + 42a + 49$

f) $(6x - 5)(6x - 5) = 36x^2 - 30x - 30x + 25$
$= 36x^2 - 60x + 25$

7. a) $x^2 + 4x + 3 = (x + 1)(x + 3)$

	x	1	1	1
x	x^2	x	x	x
1	x	1	1	1

b) $x^2 - 8x + 16 = (x - 4)^2$

	x	–1	–1	–1	–1
x	x^2	$-x$	$-x$	$-x$	$-x$
–1	$-x$	1	1	1	1
–1	$-x$	1	1	1	1
–1	$-x$	1	1	1	1
–1	$-x$	1	1	1	1

c) $3x^2 - 5x - 2 = (3x + 1)(x - 2)$

	x	-1	-1
x	x^2	$-x$	$-x$
x	x^2	$-x$	$-x$
x	x^2	$-x$	$-x$
1	x	-1	-1

d) $4x^2 - 9 = (2x + 3)(2x - 3)$

	x	x	1	1	1
x	x^2	x^2	x	x	x
x	x^2	x^2	x	x	x
-1	$-x$	$-x$	-1	-1	-1
-1	$-x$	$-x$	-1	-1	-1
-1	$-x$	$-x$	-1	-1	-1

e) $2x^2 + 12x = 2x(x + 6)$

	x	x
x	x^2	x^2
1	x	x
1	x	x
1	x	x
1	x	x
1	x	x

f) $9x^2 - 6x + 1 = (3x - 1)^2$

	x	x	x	-1
x	x^2	x^2	x^2	$-x$
x	x^2	x^2	x^2	$-x$
x	x^2	x^2	x^2	$-x$
-1	$-x$	$-x$	$-x$	1

8. a) From the graph, the zeros are –2 and 8, the y-intercept is 16, the equation of the axis of symmetry is $x = 3$, and the vertex is (3, 25). Using the zeros, the equation in factored form is $y = a(x - 8)(x + 2)$. Substitute the y-intercept into the equation to determine a:

$$\begin{aligned} y &= a(x - 8)(x + 2) \\ 16 &= a(0 - 8)(0 + 2) \\ 16 &= -16a \\ a &= -1 \end{aligned}$$

Expand $y = -(x - 8)(x + 2)$ to determine the equation in standard form.

$$\begin{aligned} y &= -(x - 8)(x + 2) \\ y &= -(x^2 + 2x - 8x - 16) \\ y &= -(x^2 - 6x - 16) \\ y &= -x^2 + 6x + 16 \end{aligned}$$

The equation in standard form is $y = -x^2 + 6x + 16$.

b) From the graph, the zeros are –6 and 2, the y-intercept is –6, the equation of the axis of symmetry is $x = -2$, and the vertex is (–2, –8). Using the zeros, the equation in factored form is $y = a(x + 6)(x - 2)$. Substitute the y-intercept into the equation to determine a:

$$\begin{aligned} y &= a(x + 6)(x - 2) \\ -6 &= a(0 + 6)(0 - 2) \\ -6 &= -12a \\ a &= \frac{1}{2} \end{aligned}$$

Expand $y = \frac{1}{2}(x + 6)(x - 2)$ to determine the equation in standard form.

$$\begin{aligned} y &= \frac{1}{2}(x + 6)(x - 2) \\ y &= (\frac{1}{2}x + 3)(x - 2) \\ y &= \frac{1}{2}x^2 - x + 3x - 6 \\ y &= \frac{1}{2}x^2 + 2x - 6 \end{aligned}$$

The equation in standard form is $y = \frac{1}{2}x^2 + 2x - 6$.

9. a) The equation is in factored form.
Substitute $x = 0$ into the equation to determine the y-intercept:

$$\begin{aligned} y &= (x-4)(x+6) \\ y &= (0-4)(0+6) \\ y &= (-4)(6) \\ y &= -24 \end{aligned}$$

The y-intercept is -24.
Use the mean of the zeros, $x = 4$ and $x = -6$, to determine the equation of the axis of symmetry:

$$\frac{4+(-6)}{2} = -1$$

The equation of the axis of symmetry is $x = -1$.
Substitute the x-coordinate of the vertex, $x = -1$, into the equation to determine the y-coordinate of the vertex:

$$\begin{aligned} y &= (x-4)(x+6) \\ y &= (-1-4)(-1+6) \\ y &= (-5)(5) \\ y &= -25 \end{aligned}$$

The vertex is $(-1, -25)$.
b) The equation is in vertex form.
Substitute $x = 0$ into the equation to determine the y-intercept:

$$\begin{aligned} y &= -4(x-3)^2 - 5 \\ y &= -4(0-3)^2 - 5 \\ y &= -4(9) - 5 \\ y &= -36 - 5 \\ y &= -41 \end{aligned}$$

The y-intercept is -41.
The equation of the axis of symmetry can be determined from the x-coordinate of the vertex, 3.
The equation of the axis of symmetry is $x = 3$.
From the equation, the vertex is $(3, -5)$.
10. a) Disagree. Some cannot be factored, e.g., $x^2 + x + 1$.
b) Agree. The graph of every quadratic relation has a vertex. The vertex shows the maximum or minimum value, e.g., x^2 has a minimum value and $-x^2$ has a maximum value.
c) Disagree. Some have no x-intercepts, e.g., $x^2 + 1$. Some have one x-intercept, e.g., $(x-3)^2$.

Lesson 6.1: Solving Quadratic Equations, page 319

1. Write each quadratic equation in the form $ax^2 + bx + c = 0$. The corresponding quadratic relation is $y = ax^2 + bx + c$.
a) The equation is given in the form $ax^2 + bx + c = 0$, so the corresponding quadratic relation is $y = x^2 - 4x + 4$.
b) Rearrange the equation so that the right side is 0:

$$\begin{aligned} 2x^2 - 9x &= 5 \\ 2x^2 - 9x - 5 &= 0 \end{aligned}$$

The corresponding quadratic relation is $y = 2x^2 - 9x - 5$.
2. a) The parabola intersects the x-axis at the points $(-3, 0)$ and $(3, 0)$, therefore the roots are -3 and 3.
b) The parabola intersects the x-axis at the points $\left(-\frac{1}{2}, 0\right)$ and $(3, 0)$, therefore the roots are $-\frac{1}{2}$ and 3.

3. a) Since $x(x+4) = 0$, either $x = 0$ or $x + 4 = 0$. Solve the second equation to obtain both solutions:

$$\begin{aligned} x + 4 &= 0 \\ x &= -4 \end{aligned}$$

The solutions are $x = 0$ and $x = -4$.
b) Since $(x+10)(x+8) = 0$, either $x + 10 = 0$ or $x + 8 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} x + 10 &= 0 & x + 8 &= 0 \\ x &= -10 & x &= -8 \end{aligned}$$

The solutions are $x = -10$ and $x = -8$.
c) Since $(x-5)^2 = 0$, it must be true that $x - 5 = 0$. The only solution is $x = 5$.
d) Since $(3x+8)(x-4) = 0$, either $3x + 8 = 0$ or $x - 4 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} 3x + 8 &= 0 & x - 4 &= 0 \\ 3x &= -8 & x &= 4 \\ x &= -\frac{8}{3} \end{aligned}$$

The solutions are $x = -\frac{8}{3}$ and $x = 4$.
e) This is a trinomial where $a = 1$. To determine its factors, determine two numbers whose product is 6 and whose sum is 5. The numbers are 2 and 3, so the factored equation is $(x+2)(x+3) = 0$. It must be true that either $x + 2 = 0$ or $x + 3 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} x + 2 &= 0 & x + 3 &= 0 \\ x &= -2 & x &= -3 \end{aligned}$$

The solutions are $x = -2$ and $x = -3$.
f) Rearrange the equation so that the right side is 0:

$$x^2 - 2x - 8 = 0$$

This is a trinomial where $a = 1$. To determine its factors, determine two numbers whose product is -8 and whose sum is -2. The numbers are 2 and -4, so the factored equation is $(x+2)(x-4) = 0$. It must be true that either $x + 2 = 0$ or $x - 4 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} x + 2 &= 0 & x - 4 &= 0 \\ x &= -2 & x &= 4 \end{aligned}$$

The solutions are $x = -2$ and $x = 4$.
4. Substitute the given value for x in the equation, and, if the equality holds, the value is a root.
a)

$$\begin{aligned} x^2 + x - 6 &= 0 \\ (2)^2 + (2) - 6 &= 0 \\ 4 + 2 - 6 &= 0 \\ 0 &= 0 \end{aligned}$$

Yes, the value $x = 2$ is a root of the equation.
b)

$$\begin{aligned} x^2 + 7x - 8 &= 0 \\ (4)^2 + 7(4) - 8 &= 0 \\ 16 + 28 - 8 &= 0 \\ 36 &= 0 \end{aligned}$$

No, the value $x = 4$ is not a root of the equation.

c)
$$2x^2+11x+5=0$$
$$2\left(-\frac{1}{2}\right)^2+11\left(-\frac{1}{2}\right)+5=0$$
$$2(0.25)-5.5+5=0$$
$$0.5-5.5+5=0$$
$$0=0$$

Yes, the value $x=-\frac{1}{2}$ is a root of the equation.

d)
$$8x^2+10x-3=0$$
$$8\left(\frac{3}{2}\right)^2+10\left(\frac{3}{2}\right)-3=0$$
$$8(2.25)+15-3=0$$
$$18+15-3=0$$
$$30=0$$

No, the value $x=\frac{3}{2}$ is not a root of the equation.

e)
$$x^2-4x-5=0$$
$$(-5)^2-4(-5)-5=0$$
$$25+20-5=0$$
$$40=0$$

No, the value $x=-5$ is not a root of the equation.

f)
$$3x^2-2x-8=0$$
$$3(2)^2-2(2)-8=0$$
$$12-4-8=0$$
$$0=0$$

Yes, the value $x=2$ is a root of the equation.

5. a) To determine this trinomial's factors, determine two numbers whose product is –15 and whose sum is 2. The numbers are 5 and –3, so the factored equation is $(x+5)(x-3)=0$. It must be true that either $x+5=0$ or $x-3=0$. Solve the two equations to obtain both solutions:

$$x+5=0 \qquad x-3=0$$
$$x=-5 \qquad x=3$$

The solutions are $x=-5$ and $x=3$.

b) To determine this trinomial's factors, determine two numbers whose product is –24 and whose sum is 5. The numbers are 8 and –3, so the factored equation is $(x+8)(x-3)=0$. It must be true that either $x+8=0$ or $x-3=0$. Solve the two equations to obtain both solutions:

$$x+8=0 \qquad x-3=0$$
$$x=-8 \qquad x=3$$

The solutions are $x=-8$ and $x=3$.

c) To determine this trinomial's factors, determine two numbers whose product is 4 and whose sum is 4. The numbers are 2 and 2, so the factored equation is $(x+2)(x+2)=0$ or $(x+2)^2=0$. It must be true that $x+2=0$, so the only solution is $x=-2$.

d) Factor out the common factor:

$$x^2-5x=0$$
$$x(x-5)=0$$

Either $x=0$ or $x-5=0$. Solve the second equation to obtain both solutions:

$$x-5=0$$
$$x=5$$

The solutions are $x=0$ and $x=5$.

e) Rearrange the equation so that the right side is 0:

$$x^2-6x=16$$
$$x^2-6x-16=0$$

To determine this trinomial's factors, determine two numbers whose product is –16 and whose sum is –6. The numbers are 2 and –8, so the factored equation is $(x+2)(x-8)=0$. It must be true that either $x+2=0$ or $x-8=0$. Solve the two equations to obtain both solutions:

$$x+2=0 \qquad x-8=0$$
$$x=-2 \qquad x=8$$

The solutions are $x=-2$ and $x=8$.

f) Rearrange the equation so that the right side is 0:

$$x^2+12=7x$$
$$x^2-7x+12=0$$

To determine this trinomial's factors, determine two numbers whose product is 12 and whose sum is –7. The numbers are –3 and –4, so the factored equation is $(x-3)(x-4)=0$. It must be true that either $x-3=0$ or $x-4=0$. Solve the two equations to obtain both solutions:

$$x-3=0 \qquad x-4=0$$
$$x=3 \qquad x=4$$

The solutions are $x=3$ and $x=4$.

6. a) Determine two numbers whose product is $(3)(-2)=-6$ and whose sum is –5. The numbers are –6 and 1. Use these to decompose the middle term and then factor:

$$3x^2-5x-2=3x^2-6x+x-2$$
$$=3x(x-2)+(x-2)$$
$$=(3x+1)(x-2)$$

Since $(3x+1)(x-2)=0$, either $3x+1=0$ or $x-2=0$. Solve the two equations to obtain both solutions:

$$3x+1=0 \qquad x-2=0$$
$$3x=-1 \qquad x=2$$
$$x=-\frac{1}{3}$$

The solutions are $x=-\frac{1}{3}$ and $x=2$.

Check:

For $x=-\frac{1}{3}$,

$$3x^2-5x-2=0$$
$$3\left(-\frac{1}{3}\right)^2-5\left(-\frac{1}{3}\right)-2=0$$
$$\frac{3}{9}+\frac{5}{3}-2=0$$
$$0=0$$

For $x=2$,

$$3x^2-5x-2=0$$
$$3(2)^2-5(2)-2=0$$
$$12-10-2=0$$
$$0=0$$

b) Determine two numbers whose product is (2)(–2) = –4 and whose sum is 3. The numbers are 4 and –1. Use these to decompose the middle term and then factor:

$$\begin{aligned} 2x^2 + 3x - 2 &= 2x^2 + 4x - x - 2 \\ &= 2x(x+2) - (x-2) \\ &= (2x-1)(x+2) \end{aligned}$$

Since $(2x-1)(x+2) = 0$, either $2x - 1 = 0$ or $x + 2 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} 2x - 1 &= 0 & x + 2 &= 0 \\ 2x &= 1 & x &= -2 \\ x &= \frac{1}{2} \end{aligned}$$

The solutions are $x = \frac{1}{2}$ and $x = -2$.

Check:

For $x = \frac{1}{2}$,

$$2x^2 + 3x - 2 = 0$$
$$2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) - 2 = 0$$
$$\frac{1}{2} + \frac{3}{2} - 2 = 0$$
$$0 = 0$$

For $x = -2$,

$$\begin{aligned} 2x^2 + 3x - 2 &= 0 \\ 2(-2)^2 + 3(-2) - 2 &= 0 \\ 8 - 6 - 2 &= 0 \\ 0 &= 0 \end{aligned}$$

c) Determine two numbers whose product is (3)(–15) = –45 and whose sum is –4. The numbers are –9 and 5. Use these to decompose the middle term and then factor:

$$\begin{aligned} 3x^2 - 4x - 15 &= 3x^2 - 9x + 5x - 15 \\ &= 3x(x-3) + 5(x-3) \\ &= (3x+5)(x-3) \end{aligned}$$

Since $(3x + 5)(x - 3) = 0$, either $3x + 5 = 0$ or $x - 3 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} 3x + 5 &= 0 & x - 3 &= 0 \\ 3x &= -5 & x &= 3 \\ x &= -\frac{5}{3} \end{aligned}$$

The solutions are $x = -\frac{5}{3}$ and $x = 3$.

Check:

For $x = -\frac{5}{3}$,

$$3x^2 - 4x - 15 = 0$$
$$3\left(-\frac{5}{3}\right)^2 - 4\left(-\frac{5}{3}\right) - 15 = 0$$
$$\frac{75}{9} + \frac{20}{3} - 15 = 0$$
$$0 = 0$$

For $x = 3$,

$$\begin{aligned} 3x^2 - 4x - 15 &= 0 \\ 3(3)^2 - 4(3) - 15 &= 0 \\ 27 - 12 - 15 &= 0 \\ 0 &= 0 \end{aligned}$$

d) Determine two numbers whose product is (6)(–2) = –12 and whose sum is –1. The numbers are –4 and 3. Use these to decompose the middle term and then factor:

$$\begin{aligned} 6x^2 - x - 2 &= 6x^2 - 4x + 3x - 2 \\ &= 2x(3x-2) + (3x-2) \\ &= (2x+1)(3x-2) \end{aligned}$$

Since $(2x + 1)(3x - 2) = 0$, either $2x + 1 = 0$ or $3x - 2 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} 2x + 1 &= 0 & 3x - 2 &= 0 \\ 2x &= -1 & 3x &= 2 \\ x &= -\frac{1}{2} & x &= \frac{2}{3} \end{aligned}$$

The solutions are $x = -\frac{1}{2}$ and $x = \frac{2}{3}$.

Check:

For $x = -\frac{1}{2}$,

$$6x^2 - x - 2 = 0$$
$$6\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) - 2 = 0$$
$$\frac{6}{4} + \frac{1}{2} - 2 = 0$$
$$0 = 0$$

For $x = \frac{2}{3}$,

$$6x^2 - x - 2 = 0$$
$$6\left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right) - 2 = 0$$
$$\frac{24}{9} - \frac{2}{3} - 2 = 0$$
$$0 = 0$$

e) Rearrange the equation so that the right side is 0:

$$\begin{aligned} 4x^2 - 4x &= 3 \\ 4x^2 - 4x - 3 &= 0 \end{aligned}$$

Determine two numbers whose product is (4)(–3) = –12 and whose sum is –4. The numbers are –6 and 2. Use these to decompose the middle term and then factor:

$$\begin{aligned} 4x^2 - 4x - 3 &= 4x^2 - 6x + 2x - 3 \\ &= 2x(2x-3) + (2x-3) \\ &= (2x+1)(2x-3) \end{aligned}$$

Since $(2x + 1)(2x - 3) = 0$, either $2x + 1 = 0$ or $2x - 3 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} 2x + 1 &= 0 & 2x - 3 &= 0 \\ 2x &= -1 & 2x &= 3 \\ x &= -\frac{1}{2} & x &= \frac{3}{2} \end{aligned}$$

The solutions are $x = -\frac{1}{2}$ and $x = \frac{3}{2}$.

Check:

For $x = -\frac{1}{2}$,

$$4x^2 - 4x - 3 = 0$$
$$4\left(-\frac{1}{2}\right)^2 - 4\left(-\frac{1}{2}\right) - 3 = 0$$
$$1 + 2 - 3 = 0$$
$$0 = 0$$

For $x = \frac{3}{2}$,

$$4x^2 - 4x - 3 = 0$$
$$4\left(\frac{3}{2}\right)^2 - 4\left(\frac{3}{2}\right) - 3 = 0$$
$$\frac{36}{4} - \frac{12}{2} - 3 = 0$$
$$0 = 0$$

f) Rearrange the equation so that the right side is 0:

$$9x^2 + 1 = 6x$$
$$9x^2 - 6x + 1 = 0$$

Since this trinomial is in the form $a^2 - 2ab + b^2$, it is a perfect square. The root of the first term is $3x$, the root of the last term is 1 and the sign on the second term is negative, therefore the factored equation is $(3x - 1)^2 = 0$. It must be true that $3x - 1 = 0$.

$$3x - 1 = 0$$
$$3x = 1$$
$$x = \frac{1}{3}$$

The only solution is $x = \frac{1}{3}$.

Check:

For $x = \frac{1}{3}$,

$$9x^2 - 6x + 1 = 0$$
$$9\left(\frac{1}{3}\right)^2 - 6\left(\frac{1}{3}\right) + 1 = 0$$
$$1 - 2 + 1 = 0$$
$$0 = 0$$

7. a) Simplify the equation by multiplying using the distributive property and then rearranging it so that the right side is 0:

$$x(x + 1) = 12$$
$$x^2 + x = 12$$
$$x^2 + x - 12 = 0$$

This is a trinomial where $a = 1$. To determine its factors, determine two numbers whose product is -12 and whose sum is 1. The numbers are -3 and 4, so the factored equation is $(x - 3)(x + 4) = 0$. It must be true that either $x - 3 = 0$ or $x - 4 = 0$. Solve the two equations to obtain both solutions:

$$x - 3 = 0 \qquad x + 4 = 0$$
$$x = 3 \qquad x = -4$$

The solutions are $x = 3$ and $x = -4$.

b) Simplify the equation by multiplying using the distributive property and then rearranging it so that the right side is 0:

$$2x(x + 4) = x + 4$$
$$2x^2 + 8x = x + 4$$
$$2x^2 + 7x - 4 = 0$$

This is a trinomial where $a \neq 1$. Determine two numbers whose product is $(2)(-4) = -8$ and whose sum is 7. The numbers are 8 and -1. Use these to decompose the middle term and then factor:

$$2x^2 + 7x - 4 = 2x^2 + 8x - x - 4$$
$$= 2x(x + 4) - (x + 4)$$
$$= (2x - 1)(x + 4)$$

Since $(2x - 1)(x + 4) = 0$, either $2x - 1 = 0$ or $x + 4 = 0$. Solve the two equations to obtain both solutions:

$$2x - 1 = 0 \qquad x + 4 = 0$$
$$2x = 1 \qquad x = -4$$
$$x = \frac{1}{2}$$

The solutions are $x = \frac{1}{2}$ and $x = -4$.

c) Simplify the equation by multiplying using the distributive property and then rearranging it so that the right side is 0:

$$3x(x + 2) = 2x^2 - (4 - x)$$
$$3x^2 + 6x = 2x^2 - 4 + x$$
$$x^2 + 5x + 4 = 0$$

This is a trinomial where $a = 1$. To determine its factors, determine two numbers whose product is 4 and whose sum is 5. The numbers are 1 and 4, so the factored equation is $(x + 1)(x + 4) = 0$. It must be true that either $x + 1 = 0$ or $x + 4 = 0$. Solve the two equations to obtain both solutions:

$$x + 1 = 0 \qquad x + 4 = 0$$
$$x = -1 \qquad x = -4$$

The solutions are $x = -1$ and $x = -4$.

d) Simplify the equation by multiplying using the distributive property and then rearranging it so that the right side is 0:

$$3x(x + 6) + 50 = 2x^2 + 3(x - 2)$$
$$3x^2 + 18x + 50 = 2x^2 + 3x - 6$$
$$x^2 + 15x + 56 = 0$$

This is a trinomial where $a = 1$. To determine its factors, determine two numbers whose product is 56 and whose sum is 15. The numbers are 7 and 8, so the factored equation is $(x + 7)(x + 8) = 0$. It must be true that either $x + 7 = 0$ or $x + 8 = 0$. Solve the two equations to obtain both solutions:

$$x + 7 = 0 \qquad x + 8 = 0$$
$$x = -7 \qquad x = -8$$

The solutions are $x = -7$ and $x = -8$.

e) Simplify the equation by multiplying using the distributive property and then rearranging it so that the right side is 0:

$$(x + 2)^2 + x = 2(3x + 5)$$
$$x^2 + 4x + 4 + x = 6x + 10$$
$$x^2 + 5x + 4 = 6x + 10$$
$$x^2 - x - 6 = 0$$

This is a trinomial where $a = 1$. To determine its factors, determine two numbers whose product is –6 and whose sum is –1. The numbers are 2 and –3, so the factored equation is $(x + 2)(x - 3) = 0$. It must be true that either $x + 2 = 0$ or $x - 3 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} x + 2 &= 0 \qquad x - 3 = 0 \\ x &= -2 \qquad\quad x = 3 \end{aligned}$$

The solutions are $x = -2$ and $x = 3$.

f) Simplify the equation by multiplying using the distributive property and then rearranging it so that the right side is 0:

$$\begin{aligned} (2x + 1)^2 &= x + 2 \\ 4x^2 + 4x + 1 &= x + 2 \\ 4x^2 + 3x - 1 &= 0 \end{aligned}$$

This is a trinomial where $a \neq 1$. Determine two numbers whose product is $(4)(-1) = -4$ and whose sum is 3. The numbers are –1 and 4. Use these to decompose the middle term and then factor:

$$\begin{aligned} 4x^2 + 3x - 1 &= 4x^2 - x + 4x - 1 \\ &= x(4x - 1) + (4x - 1) \\ &= (x + 1)(4x - 1) \end{aligned}$$

Since $(x + 1)(4x - 1) = 0$, either $x + 1 = 0$ or $4x - 1 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} x + 1 &= 0 \qquad 4x - 1 = 0 \\ x &= -1 \qquad\quad 4x = 1 \\ & \qquad\quad\; x = \frac{1}{4} \end{aligned}$$

The solutions are $x = -1$ and $x = \frac{1}{4}$.

8. a) To determine this trinomial's factors, determine two numbers whose product is –32 and whose sum is 4. The numbers are 8 and –4, so the factored equation is $(x + 8)(x - 4) = 0$. It must be true that either $x + 8 = 0$ or $x - 4 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} x + 8 &= 0 \qquad x - 4 = 0 \\ x &= -8 \qquad\quad x = 4 \end{aligned}$$

The solutions are $x = -8$ and $x = 4$.

b) To determine this trinomial's factors, determine two numbers whose product is 30 and whose sum is 11. The numbers are 6 and 5, so the factored equation is $(x + 6)(x + 5) = 0$. It must be true that either $x + 6 = 0$ or $x + 5 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} x + 6 &= 0 \qquad x + 5 = 0 \\ x &= -6 \qquad\quad x = -5 \end{aligned}$$

The solutions are $x = -6$ and $x = -5$.

c) Determine two numbers whose product is $(5)(-12) = -60$ and whose sum is –28. The numbers are –30 and 2. Use these to decompose the middle term and then factor:

$$\begin{aligned} 5x^2 - 28x - 12 &= 5x^2 - 30x + 2x - 12 \\ &= 5x(x - 6) + 2(x - 6) \\ &= (5x + 2)(x - 6) \end{aligned}$$

Since $(5x + 2)(x - 6) = 0$, either $5x + 2 = 0$ or $x - 6 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} 5x + 2 &= 0 \qquad x - 6 = 0 \\ 5x &= -2 \qquad\quad x = 6 \\ x &= -\frac{2}{5} \end{aligned}$$

The solutions are $x = -\frac{2}{5}$ and $x = 6$.

d) Rearrange the equation so that the right side is 0:

$$\begin{aligned} x^2 + 5x &= 14 \\ x^2 + 5x - 14 &= 0 \end{aligned}$$

To determine this trinomial's factors, determine two numbers whose product is –14 and whose sum is 5. The numbers are 7 and –2, so the factored equation is $(x + 7)(x - 2) = 0$. It must be true that either $x + 7 = 0$ or $x - 2 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} x + 7 &= 0 \qquad x - 2 = 0 \\ x &= -7 \qquad\quad x = 2 \end{aligned}$$

The solutions are $x = -7$ and $x = 2$.

e) Rearrange the equation so that the right side is 0:

$$\begin{aligned} 4x^2 + 25 &= 20x \\ 4x^2 - 20x + 25 &= 0 \end{aligned}$$

Since it is in the form $a^2 - 2ab + b^2$, this trinomial is a perfect square. The root of the first term is $2x$, the root of the last term is 5 and the sign on the second term is negative, therefore the factored equation is $(2x - 5)^2 = 0$. It must be true that $2x - 5 = 0$:

$$\begin{aligned} 2x - 5 &= 0 \\ 2x &= 5 \\ x &= \frac{5}{2} \end{aligned}$$

The only solution is $x = \frac{5}{2}$.

f) Rearrange the equation so that the right side is 0:

$$\begin{aligned} 3x^2 + 16x - 7 &= 5 \\ 3x^2 + 16x - 12 &= 0 \end{aligned}$$

Determine two numbers whose product is $(3)(-12) = -36$ and whose sum is 16. The numbers are 18 and –2. Use these to decompose the middle term and then factor:

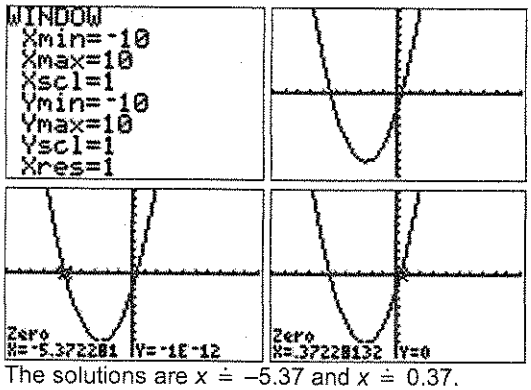
$$\begin{aligned} 3x^2 + 16x - 12 &= 3x^2 + 18x - 2x - 12 \\ &= 3x(x + 6) - 2(x + 6) \\ &= (3x - 2)(x + 6) \end{aligned}$$

Since $(3x - 2)(x + 6) = 0$, either $3x - 2 = 0$ or $x + 6 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} 3x - 2 &= 0 \qquad x + 6 = 0 \\ 3x &= 2 \qquad\quad x = -6 \\ x &= \frac{2}{3} \end{aligned}$$

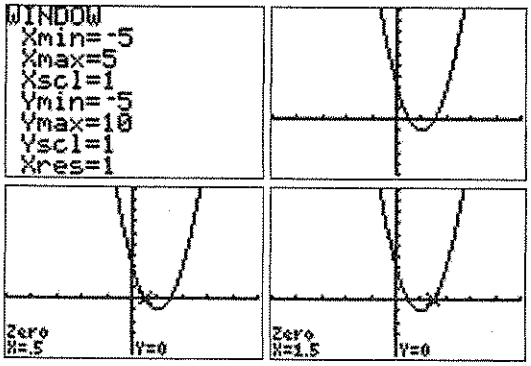
The solutions are $x = \frac{2}{3}$ and $x = -6$.

9. a) Using a graphing calculator, graph the relation $x^2 + 5x - 2 = 0$ and use the Zero operation to estimate the x-intercepts:



The solutions are $x \doteq -5.37$ and $x \doteq 0.37$.

b) Using a graphing calculator, graph the relation $4x^2 - 8x + 3 = 0$ and use the Zero operation to estimate the x-intercepts:

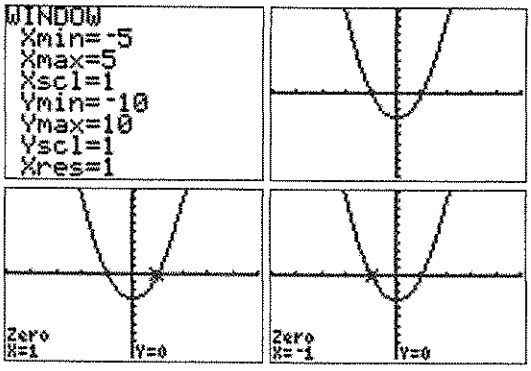


The solutions are $x = 0.5$ and $x = 1.5$.

c) Rearrange the equation so that the right side is 0:

$$x^2 + 1 = 4 - 2x^2$$
$$3x^2 - 3 = 0$$

Using a graphing calculator, graph the relation $3x^2 - 3 = 0$ and use the Zero operation to estimate the x-intercepts:

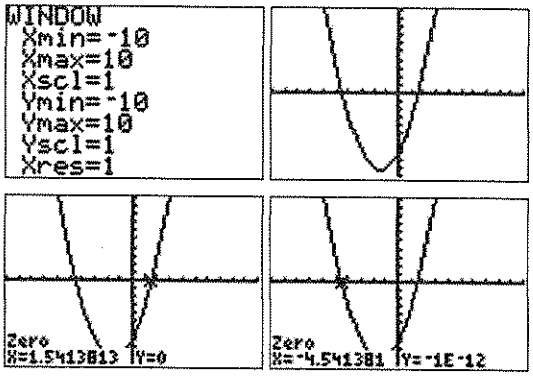


The solutions are $x = 1$ and $x = -1$.

d) First expand the terms in the equation, then rearrange it so that the right side is 0:

$$x(x + 5) = 2x + 7$$
$$x^2 + 5x = 2x + 7$$
$$x^2 + 3x - 7 = 0$$

Using a graphing calculator, graph the relation $x^2 + 3x - 7 = 0$ and use the Zero operation to estimate the x-intercepts:

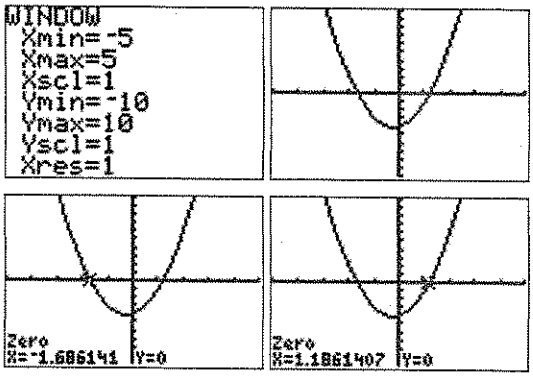


The solutions are $x \doteq 1.54$ and $x \doteq -4.54$.

e) Rearrange the equation so that the right side is 0:

$$3x^2 + 5x - 3 = x^2 + 4x + 1$$
$$2x^2 + x - 4 = 0$$

Using a graphing calculator, graph the relation $2x^2 + x - 4 = 0$ and use the Zero operation to estimate the x-intercepts:

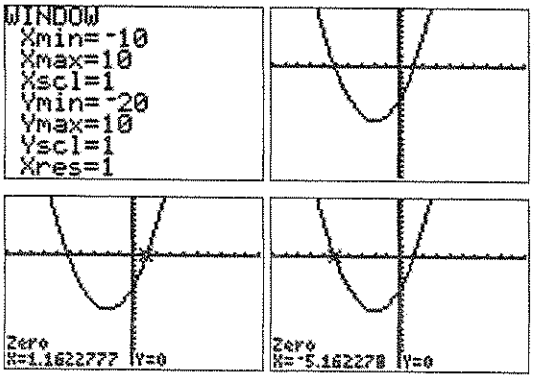


The solutions are $x \doteq -1.69$ and $x \doteq 1.19$

f) First expand and simplify the terms in the equation, then rearrange it so that the right side is 0:

$$(x + 3)^2 - 2x = 15$$
$$x^2 + 6x + 9 - 2x = 15$$
$$x^2 + 4x + 9 = 15$$
$$x^2 + 4x - 6 = 0$$

Using a graphing calculator, graph the relation $x^2 + 4x - 6 = 0$ and use the Zero operation to estimate the x-intercepts:



The solutions are $x \doteq 1.16$ and $x \doteq -5.16$.

10. a) He will break even if his profit equals zero. Let $P = 0$ and determine the roots of the resulting equation:

$$0 = -5x^2 + 200x - 1500$$

Divide out the common factor of –5 and then factor the trinomial:

$$0 = -5(x^2 - 40x + 300)$$
$$0 = -5(x - 10)(x - 30)$$

Since $-5(x - 10)(x - 30) = 0$, either $x - 10 = 0$ or $x - 30 = 0$. Solve the two equations to obtain both solutions:

$$x - 10 = 0 \qquad x - 30 = 0$$
$$x = 10 \qquad x = 30$$

The solutions are $x = 10$ and $x = 30$, therefore he needs to charge either \$10 or \$30 to break even.

b) Let $P = 500$ and then rearrange the equation so that the left side is 0:

$$500 = -5x^2 + 200x - 1500$$
$$0 = -5x^2 + 200x - 2000$$

Solve this equation to determine x, the amount he needs to charge if he is to have a profit of \$500. Divide out the common factor of –5 and then factor the trinomial:

$$0 = -5(x^2 - 40x + 400)$$
$$0 = -5(x - 20)(x - 20)$$
$$0 = -5(x - 20)^2$$

Since $-5(x - 20)^2 = 0$, it must be true that $x - 20 = 0$:

$$x - 20 = 0$$
$$x = 20$$

The only solution is $x = 20$, therefore he needs to charge \$20 to have a profit of \$500.

11. Since the area of the walkway is to be 900 m^2, let $A = 900$. Rearrange the resulting equation so that the left side is 0:

$$A = 160x + 4x^2$$
$$900 = 160x + 4x^2$$
$$0 = 4x^2 + 160x - 900$$

Divide out the common factor of 4 and then factor the trinomial:

$$0 = 4(x^2 + 40x - 225)$$
$$0 = 4(x - 5)(x + 45)$$

Since $4(x - 5)(x + 45) = 0$, either $x - 5 = 0$ or $x + 45 = 0$. Solve the two equations to obtain both solutions:

$$x - 5 = 0 \qquad x + 45 = 0$$
$$x = 5 \qquad x = -45$$

The solutions are $x = 5$ and $x = -45$, but the width of the walkway must be positive, therefore it must be 5 m.

12. a) Let $R = 900$ to determine how much he will earn when the rent is set at \$900:

$$E = \frac{1}{50}R(1650 - R)$$
$$E = \frac{1}{50}900(1650 - 900)$$
$$E = 18(750)$$
$$E = 13\,500$$

He will earn \$13 500 when the rent is set at \$900.

b) Let $E = 13\,000$ to determine the values of rent that result in earnings of \$13 000.

$$E = \frac{1}{50}R(1650 - R)$$
$$13\,000 = \frac{1}{50}R(1650 - R)$$

Multiply all the terms on both sides of the equation by 50 to eliminate the fraction, and then simplify:

$$650\,000 = R(1650 - R)$$
$$650\,000 = 1650R - R^2$$

Rearrange the equation so that the left side is 0:

$$0 = -R^2 + 1650R - 650\,000$$

Multiply all the terms on both sides of the equation by –1 to make R^2 positive and then factor the trinomial:

$$0 = R^2 - 1650R + 650\,000$$
$$0 = (R - 650)(R - 1000)$$

Since $(R - 650)(R - 1000) = 0$, either $R = 650$ or $R = 1000$. If he sets the rent at either \$650 or \$1000, he will earn exactly \$13 000, and if he sets the rent between these two values, he will earn more than \$13 000. He should set the rent between \$650 and \$1000 in order to earn at least \$13 000.

13. First, determine the x-coordinates of the points of intersection by setting the y-values of the line and the parabola equal to each other and solving for x.

$$-2x + 7 = 2x^2 + 3x - 5$$

Rearrange the equation so that the left side is 0:

$$0 = 2x^2 + 5x - 12$$

To solve this equation, determine two numbers whose product is $(2)(-12) = -24$ and whose sum is 5. The numbers are 8 and –3. Use these to decompose the middle term and then factor:

$$2x^2 + 5x - 12 = 2x^2 + 8x - 3x - 12$$
$$= 2x(x + 4) - 3(x + 4)$$
$$= (2x - 3)(x + 4)$$

Since $(2x - 3)(x + 4) = 0$, either $2x - 3 = 0$ or $x + 4 = 0$. Solve the two equations to obtain both x-coordinates:

$$2x - 3 = 0 \qquad x + 4 = 0$$
$$2x = 3 \qquad x = -4$$
$$x = 1.5$$

The x-coordinates of the points of intersection are $x = 1.5$ and $x = -4$. Substitute each into either the equation of the line or the equation of the parabola to determine the corresponding y-coordinates.

For $x = 1.5$,

$$y = -2x + 7$$
$$y = -2(1.5) + 7$$
$$y = 4$$

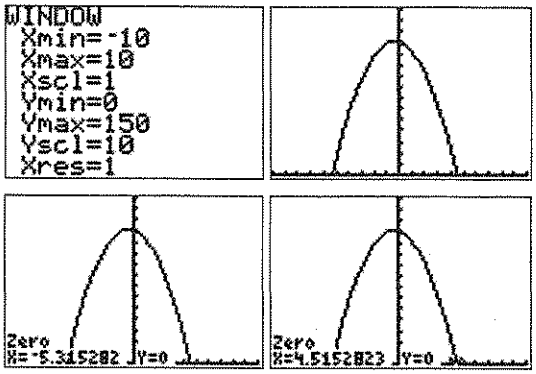
One point of intersection is (1.5, 4).

For $x = -4$,

$$y = -2x + 7$$
$$y = -2(-4) + 7$$
$$y = 15$$

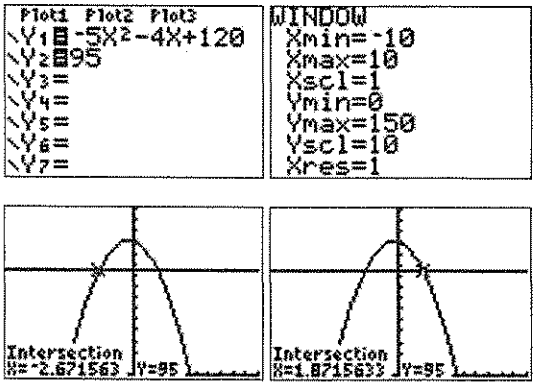
The other point of intersection is (–4, 15).

14. a) When the pebble hits the ground, its height is 0. Graph the height equation $h = -5t^2 - 4t + 120$ and locate the zeros:



The solutions are $x \doteq -5.32$ and $x \doteq 4.51$, but the time must be positive, so the time it takes the pebble to hit the ground is about 4.51 s.

b) Using a graphing calculator, graph both the height equation, $h = -5t^2 - 4t + 120$, and $h = 95$ to determine when the pebble reaches a height of 95 m. Use the intersect operation to locate the intersection points of the two graphs.



The points of intersection are $x \doteq -2.67$ and $x \doteq 1.87$, but the time must be positive, so the time it takes the pebble to fall to a height of 95 m is about 1.87 s.

15. Answers may vary, e.g., sometimes it is possible to graph the corresponding relation and solve for the zeros, either visually or by using graphing technology. Some relations, however, have no zeros. Therefore, the corresponding equations will have no solutions.

16. Answers may vary, e.g.,

a) I would rewrite any equation that had any term other than y on one side so that I could factor it. For example, $x^2 - 2x = 15$ should be rewritten as $x^2 - 2x - 15 = 0$ to make it easier to solve.

b) The x-intercepts of the relation $y = x^2 - 2x - 15$ are the solutions to the equation $x^2 - 2x - 15 = 0$.

17. First, solve the equation $x^2 - 9x + 20 = 0$ by factoring:

$$x^2 - 9x + 20 = 0$$
$$(x - 4)(x - 5) = 0$$

The solutions to $x^2 - 9x + 20 = 0$ are $x = 4$ and $x = 5$.

Notice that $x^4 - 9x^2 + 20$ is in the same form as the $x^2 - 9x + 20$, but the first term is x^4 and the second term is $-9x^2$. The factored form of $x^2 - 9x + 20$ can produce this trinomial if x is replaced by x^2:

$$\begin{aligned}(x^2 - 4)(x^2 - 5) &= x^4 - 5x^2 - 4x^2 + 20\\ &= x^4 - 9x^2 + 20\end{aligned}$$

Using this observation, the trinomial can be factored and the resulting equation solved:

$$x^4 - 9x^2 + 20 = 0$$
$$(x^2 - 4)(x^2 - 5) = 0$$

Since $(x^2 - 4)(x^2 - 5) = 0$, either $x^2 - 4 = 0$ or $x^2 - 5 = 0$.

Solve the two equations to obtain all four solutions:

$$\begin{aligned}x^2 - 4 &= 0 & x^2 - 5 &= 0\\ x^2 &= 4 & x^2 &= 5\\ x &= \pm\sqrt{4} & x &= \pm\sqrt{5}\\ x &= \pm 2\end{aligned}$$

The solutions to $x^4 - 9x^2 + 20$ are $x = \pm 2$ and $x = \pm\sqrt{5}$.

For $x^3 - 9x^2 + 20x = 0$, first factor out the common factor x:

$$x^3 - 9x^2 + 20x = 0$$
$$x(x^2 - 9x + 20) = 0$$

Factor the trinomial using the first result:

$$x(x - 4)(x - 5) = 0$$

The solutions to $x^3 - 9x^2 + 20x = 0$ are $x = 0$, $x = 4$, and $x = 5$.

18. Answers may vary, e.g., no; some equations may have two solutions, one solution, or no solutions. You can tell from the number of x-intercepts that the graph of the corresponding quadratic relation has. For example, $x^2 - 8x + 15 = 0$ has two solutions, $x^2 + 4x + 4 = 0$ has one solution, and $x^2 + 1 = 0$ has no solution.

Lesson 6.2: Exploring the Creation of Perfect Squares, page 323

1. Each trinomial is in the form $x^2 + bx + c$. To create a perfect-square trinomial, use the coefficient of x, b, to solve $c = \left(\frac{b}{2}\right)^2$.

a) The coefficient of x, b, is 8. Determine c:

$$c = \left(\frac{8}{2}\right)^2$$
$$c = 4^2$$
$$c = 16$$

The perfect-square trinomial is $x^2 + 8x + 16 = (x + 4)^2$.

b) The coefficient of x, b, is -14. Determine c:

$$c = \left(\frac{-14}{2}\right)^2$$
$$c = (-7)^2$$
$$c = 49$$

The perfect-square trinomial is $x^2 - 14x + 49 = (x - 7)^2$.

c) The coefficient of x, b, is 40. Determine c:

$$c = \left(\frac{40}{2}\right)^2$$
$$c = 20^2$$
$$c = 400$$

The perfect-square trinomial is $x^2 + 40x + 400 = (x + 20)^2$.

d) The coefficient of x, b, is 20. Determine c:

$$c = \left(\frac{20}{2}\right)^2$$

$$c = 10^2$$

$$c = 100$$

The perfect-square trinomial is
$x^2 + 20x + 100 = (x + 10)^2$.

e) The coefficient of x, b, is –5. Determine c:

$$c = \left(-\frac{5}{2}\right)^2$$

$$c = \frac{25}{4}$$

$$c = 6.25$$

The perfect-square trinomial is
$x^2 - 5x + 6.25 = (x - 2.5)^2$.

f) The coefficient of x, b, is 1. Determine c:

$$c = \left(\frac{1}{2}\right)^2$$

$$c = \frac{1}{4}$$

$$c = 0.25$$

The perfect-square trinomial is $x^2 + x + 0.25 = (x + 0.5)^2$.

2. a) First, factor a out of the expression:
$3x^2 + 30x = 3(x^2 + 10x)$
Determine c:

$$c = a\left(\frac{b}{2}\right)^2$$

$$c = 3\left(\frac{10}{2}\right)^2$$

$$c = 3(5)^2$$

$$c = 75$$

b) First, factor a out of the expression:
$2x^2 - 12x = 2(x^2 - 6x)$
Determine c:

$$c = a\left(\frac{b}{2}\right)^2$$

$$c = a\left(\frac{-6}{2}\right)^2$$

$$c = 2(-3)^2$$

$$c = 18$$

c) First, factor a out of the expression:
$-4x^2 - 8x = -4(x^2 + 2x)$
Determine c:

$$c = a\left(\frac{b}{2}\right)^2$$

$$c = -4\left(\frac{2}{2}\right)^2$$

$$c = -4(1)^2$$

$$c = -4$$

d) First, factor a out of the expression:
$6x^2 - 60x = 6(x^2 - 10x)$
Determine c:

$$c = a\left(\frac{b}{2}\right)^2$$

$$c = 6\left(\frac{-10}{2}\right)^2$$

$$c = 6(-5)^2$$

$$c = 150$$

e) First, factor a out of the expression:
$5x^2 - 10x = 5(x^2 - 2x)$
Determine c:

$$c = a\left(\frac{b}{2}\right)^2$$

$$c = 5\left(\frac{-2}{2}\right)^2$$

$$c = 5(-1)^2$$

$$c = 5$$

f) First, factor a out of the expression:
$7x^2 + 42x = 7(x^2 + 6x)$
Determine c:

$$c = a\left(\frac{b}{2}\right)^2$$

$$c = 7\left(\frac{6}{2}\right)^2$$

$$c = 7(3)^2$$

$$c = 63$$

Curious Math, page 324

1. 1) Begin with the statement $a = b$.
2) Next, multiply each side by a: $a \times a = a \times b$
3) Simplify the equation: $a^2 = ab$
4) Subtract b^2 from each side: $a^2 - b^2 = ab - b^2$
5) Factor each side: $(a + b)(a - b) = b(a - b)$
6) Divide each side by $(a - b)$: $a + b = b$
7) Substitute $a = 1$ and $b = 1$: $2 = 1$
8) Add 63 to each side: $2 + 63 = 1 + 63$
9) The final result is $65 = 64$

2. In step 6 each side of the equation is divided by $(a - b)$, but since $a = b$ the equation is being divided by zero.

3. Each shape of the same colour appears to be the same size, and since all of the colours are present in both the rectangle and the square, it appears that the rectangle and the square have the same area.

4. The rectangle is 5 units by 13 units and the square is 8 units by 8 units.
Area of rectangle $= 5 \times 13$
$= 65$
Area of square $= 8 \times 8$
$= 64$

5. It appears that the area of the rectangle equals the area of the square, and so it appears that $65 = 64$.

6. By changing the number that is added to the equation in step 8, every number could be proven equal to the next number. This would prove that all numbers are equal. This would contradict many results in mathematics and also make numbers used in our daily lives, such as distances, weights, times, and temperatures, meaningless.

7. Answers may vary, e.g.,

1) Begin with the statement $-1 = -1$

2) Write both sides as fractions: $\frac{1}{-1} = \frac{-1}{1}$

3) Take the square root of both sides: $\sqrt{\frac{1}{-1}} = \sqrt{\frac{-1}{1}}$

4) Simplify the equation using rules of square roots: $\frac{\sqrt{1}}{\sqrt{-1}} = \frac{\sqrt{-1}}{\sqrt{1}}$

5) Multiply: $\sqrt{1} \times \sqrt{1} = \sqrt{-1} \times \sqrt{-1}$

6) Any number's square root multiplied by itself gives the original number. Use this fact to simplify the equation and obtain the final result: $1 = -1$

The error in the proof is in step 4. The rule for simplifying square roots that states $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$ is only generally valid when x and y are both positive numbers.

Lesson 6.3: Completing the Square, page 331

1. a) $y = x^2 + 12x + 5$
$y = x^2 + 12x + 36 - 36 + 5$
$y = (x^2 + 12x + 36) - 36 + 5$
$y = (x + 6)^2 - 31$

Therefore ■ = 36, ◆ = 6, and ● = 31.

b) $y = 4x^2 + 24x - 15$
$y = 4(x^2 + 6x) - 15$
$y = 4(x^2 + 6x + 9 - 9) - 15$
$y = 4[(x^2 + 6x + 9) - 9] - 15$
$y = 4(x + 3)^2 - 36 - 15$
$y = 4(x + 3)^2 - 51$

Therefore ■ = 6, ◆ = 9, ● = 3, ⬟ = 36, and ★ = 51.

2. a) The square of half of the coefficient of the x term is $\left(\frac{8}{2}\right)^2 = 16$. Add and subtract this to the relation and then factor to complete the square:

$y = x^2 + 8x$
$y = x^2 + 8x + 16 - 16$
$y = (x + 4)^2 - 16$

b) The square of half of the coefficient of the x term is $\left(-\frac{12}{2}\right)^2 = 36$. Add and subtract this to the relation and then factor to complete the square:

$y = x^2 - 12x - 3$
$y = x^2 - 12x + 36 - 36 - 3$
$y = (x - 6)^2 - 39$

c) The square of half of the coefficient of the x term is $\left(\frac{8}{2}\right)^2 = 16$. Add and subtract this to the relation and then factor to complete the square:

$y = x^2 + 8x + 6$
$y = x^2 + 8x + 16 - 16 + 6$
$y = (x + 4)^2 - 10$

3. a) First, factor out 2:

$y = 2x^2 + 8x$
$y = 2(x^2 + 4x)$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is $\left(\frac{4}{2}\right)^2 = 4$:

$y = 2(x^2 + 4x + 4 - 4)$

Finish completing the square by factoring the trinomial and then simplifying:

$y = 2[(x + 2)^2 - 4]$
$y = 2(x + 2)^2 - 8$

The vertex is (–2, –8).

b) First, factor out –5 from the first two terms:

$y = -5x^2 - 20x + 6$
$y = -5(x^2 + 4x) + 6$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is $\left(\frac{4}{2}\right)^2 = 4$:

$y = -5(x^2 + 4x + 4 - 4) + 6$

Finish completing the square by factoring the trinomial and then simplifying:

$y = -5[(x + 2)^2 - 4] + 6$
$y = -5(x + 2)^2 + 20 + 6$
$y = -5(x + 2)^2 + 26$

The vertex is (–2, 26).

c) First, factor out 4 from the first two terms:

$y = 4x^2 - 10x + 1$
$= 4(x^2 - 2.5x) + 1$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is $\left(\frac{2.5}{2}\right)^2 = 1.5625$:

$y = 4(x^2 - 2.5x + 1.5625 - 1.5625) + 1$

Finish completing the square by factoring the trinomial and then simplifying:

$y = 4[(x - 1.25)^2 - 1.5625] + 1$
$y = 4(x - 1.25)^2 - 6.25 + 1$
$y = 4(x - 1.25)^2 - 5.25$

The vertex is (1.25, –5.25).

4. a) First, factor out –2 from the first two terms:

$y = -2x^2 + 12x - 11$
$y = -2(x^2 - 6x) - 11$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is $\left(-\frac{6}{2}\right)^2 = 9$:

$y = -2(x^2 - 6x + 9 - 9) - 11$

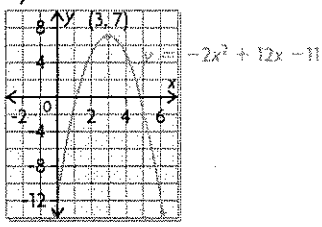
Finish completing the square by factoring the trinomial and then simplifying:

$y = -2[(x-3)^2 - 9] - 11$

$y = -2(x-3)^2 + 18 - 11$

$y = -2(x-3)^2 + 7$

b)



5. a) The square of half of the coefficient of the x term is $\left(\frac{14}{2}\right)^2 = 49$. Add and subtract this to the relation and then factor to complete the square:

$y = x^2 + 14x$

$y = x^2 + 14x + 49 - 49$

$y = (x+7)^2 - 49$

Since $a > 0$, the y-coordinate of the vertex is the minimum value of the relation. The minimum value is -49.

b) First, factor out 8 from the first two terms:

$y = 8x^2 - 96x + 15$

$y = 8(x^2 - 12x) + 15$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is $\left(-\frac{12}{2}\right)^2 = 36$:

$y = 8(x^2 - 12x + 36 - 36) + 15$

Finish completing the square by factoring the trinomial and then simplifying:

$y = 8[(x-6)^2 - 36] + 15$

$y = 8(x-6)^2 - 288 + 15$

$y = 8(x-6)^2 - 273$

Since $a > 0$, the y-coordinate of the vertex is the minimum value of the relation. The minimum value is -273.

c) First, factor out -12 from the first two terms:

$y = -12x^2 + 96x + 6$

$y = -12(x^2 - 8x) + 6$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is $\left(-\frac{8}{2}\right)^2 = 16$:

$y = -12(x^2 - 8x + 16 - 16) + 6$

Finish completing the square by factoring the trinomial and then simplifying:

$y = -12[(x-4)^2 - 16] + 6$

$y = -12(x-4)^2 + 192 + 6$

$y = -12(x-4)^2 + 198$

Since $a < 0$, the y-coordinate of the vertex is the maximum value of the relation. The maximum value is 198.

d) First, factor out -10 from the first two terms:

$y = -10x^2 + 20x - 5$

$y = -10(x^2 - 2x) - 5$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is $\left(-\frac{2}{2}\right)^2 = 1$:

$y = -10(x^2 - 2x + 1 - 1) - 5$

Finish completing the square by factoring the trinomial and then simplifying:

$y = -10[(x-1)^2 - 1] - 5$

$y = -10(x-1)^2 + 10 - 5$

$y = -10(x-1)^2 + 5$

Since $a < 0$, the y-coordinate of the vertex is the maximum value of the relation. The maximum value is 5.

e) First, factor out -4.9 from the first two terms:

$y = -4.9x^2 - 19.6x + 0.5$

$y = -4.9(x^2 + 4x) + 0.5$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is $\left(\frac{4}{2}\right)^2 = 4$:

$y = -4.9(x^2 + 4x + 4 - 4) + 0.5$

Finish completing the square by factoring the trinomial and then simplifying:

$y = -4.9[(x+2)^2 - 4] + 0.5$

$y = -4.9(x+2)^2 + 19.6 + 0.5$

$y = -4.9(x+2)^2 + 20.1$

Since $a < 0$, the y-coordinate of the vertex is the maximum value of the relation. The maximum value is 20.1.

f) First, factor out 2.8 from the first two terms:

$y = 2.8x^2 - 33.6x + 3.1$

$y = 2.8(x^2 - 12x) + 3.1$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is $\left(-\frac{12}{2}\right)^2 = 36$:

$y = 2.8(x^2 - 12x + 36 - 36) + 3.1$

Finish completing the square by factoring the trinomial and then simplifying:

$y = 2.8[(x-6)^2 - 36] + 3.1$

$y = 2.8(x-6)^2 - 100.8 + 3.1$

$y = 2.8(x-6)^2 - 97.7$

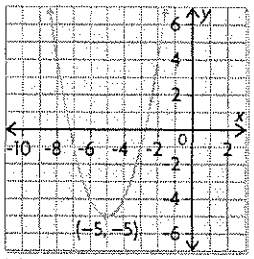
Since $a > 0$, the y-coordinate of the vertex is the minimum value of the relation. The minimum value is -97.7.

6. a) The square of half of the coefficient of the x term is $\left(\frac{10}{2}\right)^2 = 25$. Add and subtract this to the relation and then factor to complete the square:

$y = x^2 + 10x + 20$

$y = x^2 + 10x + 25 - 25 + 20$

$y = (x+5)^2 - 5$



b) First, factor out –1 from the first two terms:

$y = -x^2 + 6x - 1$

$y = -(x^2 - 6x) - 1$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is

$\left(\frac{6}{2}\right)^2 = 9$:

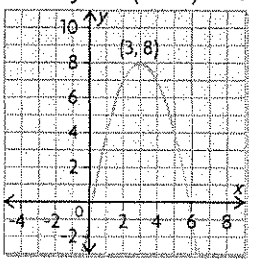
$y = -(x^2 - 6x + 9 - 9) - 1$

Finish completing the square by factoring the trinomial and then simplifying:

$y = -[(x - 3)^2 - 9] - 1$

$y = -(x - 3)^2 + 9 - 1$

$y = -(x - 3)^2 + 8$



c) First, factor out 2 from the first two terms:

$y = 2x^2 + 4x - 2$

$y = 2(x^2 + 2x) - 2$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is

$\left(\frac{2}{2}\right)^2 = 1$:

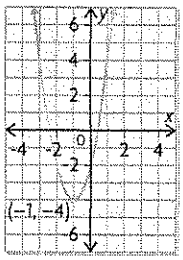
$y = 2(x^2 + 2x + 1 - 1) - 2$

Finish completing the square by factoring the trinomial and then simplifying:

$y = 2[(x + 1)^2 - 1] - 2$

$y = 2(x + 1)^2 - 2 - 2$

$y = 2(x + 1)^2 - 4$



d) First, factor out –0.5 from the first two terms:

$y = -0.5x^2 - 3x + 4$

$y = -0.5(x^2 + 6x) + 4$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is

$\left(\frac{6}{2}\right)^2 = 9$:

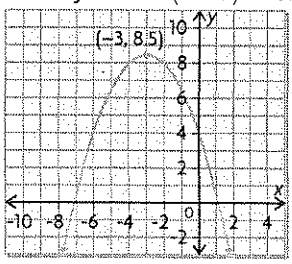
$y = -0.5(x^2 + 6x + 9 - 9) + 4$

Finish completing the square by factoring the trinomial and then simplifying:

$y = -0.5[(x + 3)^2 - 9] + 4$

$y = -0.5(x + 3)^2 + 4.5 + 4$

$y = -0.5(x + 3)^2 + 8.5$



7. a) The square of half of the coefficient of the x term is $\left(-\frac{8}{2}\right)^2 = 16$. Add and subtract this to the relation and then factor to complete the square:

$y = x^2 - 8x + 4$

$y = x^2 - 8x + 16 - 16 + 4$

$y = (x - 4)^2 - 12$

$a = 1$ and the vertex is $(4, -12)$, so the transformation that must be applied to x^2 to graph the relation is a translation 4 units right and 12 units down.

b) The trinomial is a perfect square. Factor:

$y = x^2 + 12x + 36$

$y = (x + 6)^2$

$a = 1$ and the vertex is $(-6, 0)$, so the transformation that must be applied to x^2 to graph the relation is a translation 6 units left.

c) First, factor out 4 from the first two terms:

$y = 4x^2 + 16x + 36$

$y = 4(x^2 + 4x) + 36$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is

$\left(\frac{4}{2}\right)^2 = 4$:

$y = 4(x^2 + 4x + 4 - 4) + 36$

Finish completing the square by factoring the trinomial and then simplifying:

$y = 4[(x + 2)^2 - 4] + 36$

$y = 4(x + 2)^2 - 16 + 36$

$y = 4(x + 2)^2 + 20$

Since $a = 4$ and the vertex is $(-2, 20)$, the transformations that must be applied to x^2 to graph the relation are a vertical stretch by a factor of 4 and a translation 2 units left and 20 units up.

d) First, factor out –3 from the first two terms:

$y = -3x^2 + 12x - 6$

$y = -3(x^2 - 4x) - 6$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is $\left(-\frac{4}{2}\right)^2 = 4$:

$y = -3(x^2 - 4x + 4 - 4) - 6$

Finish completing the square by factoring the trinomial and then simplifying:

$y = -3[(x-2)^2 - 4] - 6$

$y = -3(x-2)^2 + 12 - 6$

$y = -3(x-2)^2 + 6$

Since $a = -3$ and the vertex is (2, 6), the transformations that must be applied to x^2 to graph the relation are a vertical stretch by a factor of 3, a reflection in the x-axis, and a translation 2 units right and 6 units up.

e) First, factor out 0.5 from the first two terms:

$y = 0.5x^2 - 4x - 8$

$y = 0.5(x^2 - 8x) - 8$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is $\left(-\frac{8}{2}\right)^2 = 16$:

$y = 0.5(x^2 - 8x + 16 - 16) - 8$

Finish completing the square by factoring the trinomial and then simplifying:

$y = 0.5[(x-4)^2 - 16] - 8$

$y = 0.5(x-4)^2 - 8 - 8$

$y = 0.5(x-4)^2 - 16$

Since $a = 0.5$ and the vertex is (4, –16), the transformations that must be applied to x^2 to graph the relation are a vertical compression by a factor of 0.5, and a translation 4 units right and 16 units down.

f) First, factor out 2 from the first two terms:

$y = 2x^2 - x + 3$

$y = 2\left(x^2 - \frac{1}{2}x\right) + 3$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is $\left(-\frac{\frac{1}{2}}{2}\right)^2 = \frac{1}{16}$:

$y = 2\left(x^2 - \frac{1}{2}x + \frac{1}{16} - \frac{1}{16}\right) + 3$

Finish completing the square by factoring the trinomial and then simplifying:

$y = 2\left[\left(x - \frac{1}{4}\right)^2 - \frac{1}{16}\right] + 3$

$y = 2\left(x - \frac{1}{4}\right)^2 - \frac{1}{8} + 3$

$y = 2\left(x - \frac{1}{4}\right)^2 + \frac{23}{8}$

Since $a = 2$ and the vertex is $\left(\frac{1}{4}, \frac{23}{8}\right)$, so the transformations that must be applied to x^2 to graph the relation are a vertical stretch by a factor of 2 and a translation 0.25 units right and 2.875 units up.

8. a) There is no constant term in the equation, so at the moment Joan kicks the ball, $x = 0$ and $h = 0$. This indicates that Joan kicked the ball from a height of 0 m.

b) $h = -1.2x^2 + 6x$

$h = -1.2(x^2 - 5x)$

$h = -1.2(x^2 - 5x + 6.25 - 6.25)$

$h = -1.2[(x-2.5)^2 - 6.25]$

$h = -1.2(x-2.5)^2 + 7.5$

c) The vertex of the relation is (2.5, 7.5).

d) The x-coordinate of the vertex, 2.5, represents the horizontal distance, in metres, from Joan where the ball reached its maximum height. The y-coordinate of the vertex, 7.5, represents the maximum height of the ball in metres.

e) Since the ball started on the ground and ended on the ground and the vertex is halfway between those two points, Joan kicked the ball $2 \times 2.5 = 5$ m.

9. The x-coordinate of the vertex represents the number of clients that results in the maximum profit. Determine the x-coordinate of the vertex by completing the square:

$P = -15x^2 + 240x - 640$

$P = -15(x^2 - 16x) - 640$

$P = -15(x^2 - 16x + 64 - 64) - 640$

$P = -15[(x-8)^2 - 64] - 640$

$P = -15(x-8)^2 + 960 - 640$

$P = -15(x-8)^2 + 320$

The x-coordinate of the vertex is 8, so she should book 8 clients to maximize her profit.

10. The x-coordinate of the vertex represents the number of landscapers that results in the least cost. Determine the x-coordinate of the vertex by completing the square:

$C = 6x^2 - 60x + 900$

$C = 6(x^2 - 10x) + 900$

$C = 6(x^2 - 10x + 25 - 25) + 900$

$C = 6[(x-5)^2 - 25] + 900$

$C = 6(x-5)^2 - 150 + 900$

$C = 6(x-5)^2 + 750$

The x-coordinate of the vertex is 5, so to minimize cost, 5 landscapers should be hired.

11. $y = -2x^2 + 16 - 7$: no error

$y = -2(x^2 + 8x) - 7$: $+8x$ should be $-8x$

$y = -2(x^2 + 8x + 64 - 64) - 7$: $+64 - 64$ should be $+16 - 16$

$y = -2(x+8)^2 - 64 - 7$: -64 should be $-64 \times (-2)$

$y = -2(x+8)^2 - 73$: no error

Therefore, the vertex is at (73, –8): (73, –8) should be (–8, 73).

The correct solution is

$y = -2x^2 + 16x - 7$

$y = -2(x^2 - 8x + 16 - 16) - 7$

$y = -2(x-4)^2 + 32 - 7$

$y = -2(x-4)^2 + 25$

The vertex is at (4, 25).

12. The length of one piece of the wire will be the distance Bob cuts it from the end. Let this distance be x. The length of the other piece of wire will be the distance of this cut from the other end of the wire, or $60 - x$. The area of the two squares will be x^2 and $(60 - x)^2$. Add these two areas together and then simplify to determine an expression for the total area, A:

$A = x^2 + (60 - x)^2$
$A = x^2 + 3600 - 120x + x^2$
$A = 2x^2 - 120x + 3600$

Complete the square to determine the length of one piece of wire, x, that results in the minimum total area. This is represented by the x-coordinate of the vertex:

$A = 2(x^2 - 60x) + 3600$
$A = 2(x^2 - 60x + 900 - 900) + 3600$
$A = 2[(x - 30)^2 - 900] + 3600$
$A = 2(x - 30)^2 - 1800 + 3600$
$A = 2(x - 30)^2 + 1800$

To minimize the total area, the length of one piece of wire, x, should be 30 cm. The length of the other piece of wire, $60 - x$, should be $60 - (30) = 30$ cm. Therefore, each piece should be 30 cm.

13. Let the points where the bridge meets the banks be the zeros of the equation. These two points are (0, 0) and (200, 0). Use the factored form of a quadratic equation to model the bridge:

$y = a(x - 0)(x - 200)$
$y = ax(x - 200)$

The bridge is to be 51 cm high at a point 30 cm horizontally from the bank. Use this information to solve for a:

$y = ax(x - 200)$
$51 = a(30)(30 - 200)$
$51 = -5100a$
$a = -0.01$

The equation that models the height of the bridge is $y = -0.01x(x - 200)$. The x-coordinate of the vertex is halfway between the two zeros, 0 and 200. Therefore, it is 100 cm, and the maximum height of the bridge will be where $x = 100$ cm. Substitute $x = 100$ into the equation that models the bridge to determine what the maximum height is:

$y = -0.01x(x - 200)$
$y = -0.01(100)(100 - 200)$
$y = -0.01(-10\,000)$
$y = 100$

The maximum height is 100 cm.

14. a) $y = 2x^2 - 4x + 5$
$y = 2(x^2 - 2x) + 5$
$y = 2(x^2 - 2x + 1 - 1) + 5$
$y = 2[(x - 1)^2 - 1] + 5$
$y = 2(x - 1)^2 - 2 + 5$
$y = 2(x - 1)^2 + 3$

The vertex is (1, 3).

b) Answers may vary, e.g., changing the value of the constant term would not affect the x-coordinate of the vertex, but it would increase or decrease the y-coordinate of the vertex by the amount that 3 is increased or decreased by. So an increase in the constant term would result in a vertical translation of the entire parabola.

15. Complete the square for the equation that models Tammy's height above the river to determine her minimum height. If this minimum height is less than 80 cm, Tammy should not jump.

$h = 12x^2 - 76.8x + 198$
$h = 12(x^2 - 6.4x) + 198$
$h = 12(x^2 - 6.4x + 10.24 - 10.24) + 198$
$h = 12[(x - 3.2)^2 - 10.24] + 198$
$h = 12(x - 3.2)^2 - 122.88 + 198$
$h = 12(x - 3.2)^2 + 75.12$

The vertex is (3.2, 75.12), so Tammy would reach a minimum height of 75.12 cm if she jumped. She should not jump since she would be within 80 cm of the crocodile.

16. Answers may vary, e.g.,

Strategy 1: Factor the equation, and then use the factors to determine the zeros. Use the zeros to determine the equation of the axis of symmetry. Substitute into the equation to determine the y-coordinate of the vertex.

Strategy 2: Complete the square to write the equation in vertex form.

Strategy 3: Use partial factoring to determine two points with a y-coordinate of -35. Use the midpoint formula to determine a point on the axis of symmetry. Substitute the x-coordinate of the midpoint into the equation to determine the maximum or minimum value. I prefer completing the square because it gives the vertex directly.

17. Since her maximum profit will be obtained if she grooms 11 dogs, the vertex of the quadratic relation that models her profit will be $(11, p)$, where p represents her maximum profit. Use the vertex form of a quadratic equation to model her profit: $y = a(x - 11)^2 + p$, where x is the number of dogs groomed and y is the profit.

When she groomed 4 dogs, she had a profit of \$26.80, so substitute $x = 4$, $y = 26.80$ into the equation and rearrange to solve for p:

$26.80 = a(4 - 11)^2 + p$
$26.80 = 49a + p$
$p = -49a + 26.80$

When she groomed 15 dogs, she had a profit of \$416.20, so substitute $x = 15$, $y = 416.20$ into the equation and rearrange to solve for p:

$416.20 = a(15 - 11)^2 + p$
$416.20 = 16a + p$
$p = -16a + 416.20$

Use the two equations for p to solve for a by setting them equal to each other:

$-49a + 26.80 = -16a + 416.20$
$-33a = 389.40$
$a = -11.8$

Now, substitute this value for a into one of the equations for p to determine the maximum profit:

$p = -49a + 26.80$
$p = -49(-11.8) + 26.80$
$p = 578.20 + 26.80$
$p = 605$

Her maximum profit is \$605.00.

18. $y = x^2 + bx + c$

$$y = x^2 + bx + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

$$y = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

$$y = \left(x + \frac{b}{2}\right)^2 - \frac{b^2}{4} + c$$

The vertex is $(-0.5b, -0.25b^2 + c)$.

Mid-Chapter Review, page 335

1. a) Rearrange the equation so that the right side is 0:

$x^2 + 4x = 12$

$x^2 + 4x - 12 = 0$

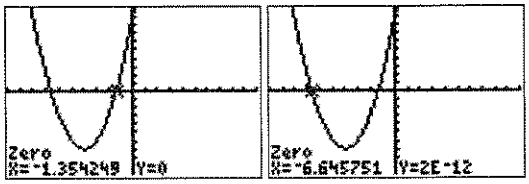
To determine this trinomial's factors, determine two numbers whose product is –12 and whose sum is 4. The numbers are –2 and 6, so the factored equation is $(x - 2)(x + 6) = 0$. It must be true that either $x - 2 = 0$ or $x + 6 = 0$. Solve the two equations to obtain both solutions:

$x - 2 = 0$ $\quad$ $x + 6 = 0$

$x = 2$ $\quad$ $x = -6$

The solutions are $x = 2$ and $x = -6$.

b) $x^2 + 8x + 9$ cannot be factored over the integers. Graph the equation to determine the x-intercepts:



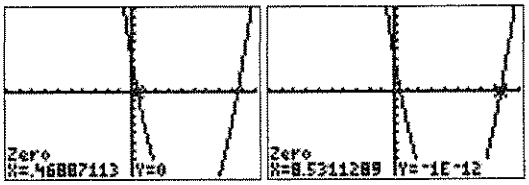
The solutions are $x \doteq -1.35$ and $x \doteq -6.65$.

c) Rearrange the equation so that the right side is 0:

$x^2 - 9x = -4$

$x^2 - 9x + 4 = 0$

$x^2 - 9x + 4$ cannot be factored over the integers. Graph the equation to determine the x-intercepts.



The solutions are $x \doteq 0.47$ and $x \doteq 8.53$.

d) $-3x^2 - 2x + 3$ cannot be factored over the integers. Graph the equation to determine the x-intercepts.

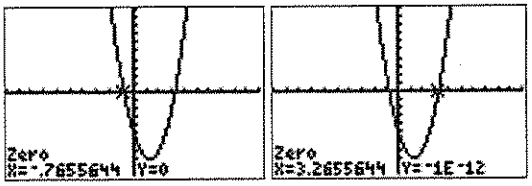
The solutions are $x \doteq -1.39$ and $x \doteq 0.72$.

e) Rearrange the equation so that the right side is 0:

$2x^2 - 5x + 10 = 15$

$2x^2 - 5x - 5 = 0$

$2x^2 - 5x - 5$ cannot be factored over the integers. Graph the equation to determine the x-intercepts.



The solutions are $x \doteq -0.77$ and $x \doteq 3.27$.

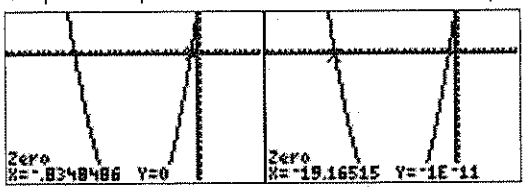
f) Rearrange the equation so that the right side is 0:

$\frac{1}{2}x^2 + 10x - 2 = -10$

$\frac{1}{2}x^2 + 10x + 8 = 0$

$\frac{1}{2}x^2 + 10x + 8$ cannot be factored over the integers.

Graph the equation to determine the x-intercepts.



The solutions are $x \doteq -0.83$ and $x \doteq -19.17$.

2. a) To determine this trinomial's factors, determine two numbers whose product is –16 and whose sum is 6. The numbers are 8 and –2, so the factored equation is $(x + 8)(x - 2) = 0$. It must be true that either $x + 8 = 0$ or $x - 2 = 0$. Solve the two equations to obtain both solutions:

$x + 8 = 0$ $\quad$ $x - 2 = 0$

$x = -8$ $\quad$ $x = 2$

The solutions are $x = -8$ and $x = 2$.

b) Determine two numbers whose product is $(2)(-3) = -6$ and whose sum is 1. The numbers are –2 and 3. Use these to decompose the middle term and then factor:

$$\begin{aligned} 2x^2 + x - 3 &= 2x^2 - 2x + 3x - 3 \\ &= 2x(x - 1) + 3(x - 1) \\ &= (2x + 3)(x - 1) \end{aligned}$$

Since $(2x + 3)(x - 1) = 0$, either $2x + 3 = 0$ or $x - 1 = 0$. Solve the two equations to obtain both solutions:

$2x + 3 = 0$ $\quad$ $x - 1 = 0$

$2x = -3$ $\quad$ $x = 1$

$x = -\frac{3}{2}$

The solutions are $x = -\frac{3}{2}$ and $x = 1$.

c) To determine this trinomial's factors, determine two numbers whose product is –10 and whose sum is 3. The numbers are 5 and –2, so the factored equation is $(x + 5)(x - 2) = 0$. It must be true that either $x + 5 = 0$ or $x - 2 = 0$. Solve the two equations to obtain both solutions:

$x + 5 = 0$ $\quad$ $x - 2 = 0$

$x = -5$ $\quad$ $x = 2$

The solutions are $x = -5$ and $x = 2$.

d) Determine two numbers whose product is $(6)(-5) = -30$ and whose sum is 7. The numbers are –3 and 10.

Use these to decompose the middle term and then factor:

$$\begin{aligned} 6x^2 + 7x - 5 &= 6x^2 - 3x + 10x - 5 \\ &= 3x(2x - 1) + 5(2x - 1) \\ &= (3x + 5)(2x - 1) \end{aligned}$$

Since $(3x + 5)(2x - 1) = 0$, either $3x + 5 = 0$ or $2x - 1 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} 3x + 5 &= 0 & 2x - 1 &= 0 \\ 3x &= -5 & 2x &= 1 \\ x &= -\frac{5}{3} & x &= \frac{1}{2} \end{aligned}$$

The solutions are $x = -\frac{5}{3}$ and $x = \frac{1}{2}$.

e) First, factor out the common factor -3:

$$\begin{aligned} -3x^2 - 9x + 12 &= 0 \\ -3(x^2 + 3x - 4) &= 0 \end{aligned}$$

To determine this trinomial's factors, determine two numbers whose product is -4 and whose sum is 3. The numbers are 4 and -1, so the factored equation is $-3(x + 4)(x - 1) = 0$. It must be true that either $x + 4 = 0$ or $x - 1 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} x + 4 &= 0 & x - 1 &= 0 \\ x &= -4 & x &= 1 \end{aligned}$$

The solutions are $x = -4$ and $x = 1$.

f) First, factor out the common factor $\frac{1}{2}$:

$$\frac{1}{2}x^2 + 6x + 16 = 0$$

$$\frac{1}{2}\left(x^2 + 12x + 32\right) = 0$$

To determine this trinomial's factors, determine two numbers whose product is 32 and whose sum is 12. The numbers are 8 and 4, so the factored equation is $\frac{1}{2}(x + 8)(x + 4) = 0$. It must be true that either $x + 8 = 0$ or $x + 4 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} x + 8 &= 0 & x + 4 &= 0 \\ x &= -8 & x &= -4 \end{aligned}$$

The solutions are $x = -8$ and $x = -4$.

3. a) Rearrange the equation so that the right side is 0:

$$\begin{aligned} x^2 + 12x + 45 &= 10 \\ x^2 + 12x + 35 &= 0 \end{aligned}$$

To determine this trinomial's factors, determine two numbers whose product is 35 and whose sum is 12. The numbers are 7 and 5, so the factored equation is $(x + 7)(x + 5) = 0$. It must be true that either $x + 7 = 0$ or $x + 5 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} x + 7 &= 0 & x + 5 &= 0 \\ x &= -7 & x &= -5 \end{aligned}$$

The solutions are $x = -7$ and $x = -5$.

b) Rearrange the equation so that the right side is 0:

$$\begin{aligned} 2x^2 + 7x + 5 &= 9 \\ 2x^2 + 7x - 4 &= 0 \end{aligned}$$

Determine two numbers whose product is $(2)(-4) = -8$ and whose sum is 7. The numbers are 8 and -1.

Use these to decompose the middle term and then factor:

$$\begin{aligned} 2x^2 + 7x - 4 &= 2x^2 + 8x - x - 4 \\ &= 2x(x + 4) - (x + 4) \\ &= (2x - 1)(x + 4) \end{aligned}$$

Since $(2x - 1)(x + 4) = 0$, either $2x - 1 = 0$ or $x + 4 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} 2x - 1 &= 0 & x + 4 &= 0 \\ 2x &= 1 & x &= -4 \\ x &= \frac{1}{2} \end{aligned}$$

The solutions are $x = \frac{1}{2}$ and $x = -4$.

c) Simplify using the distributive property and then rearrange the equation so that the right side is 0:

$$\begin{aligned} x(6x - 1) &= 12 \\ 6x^2 - x &= 12 \\ 6x^2 - x - 12 &= 0 \end{aligned}$$

Determine two numbers whose product is $(6)(-12) = -72$ and whose sum is -1. The numbers are -9 and 8. Use these to decompose the middle term and then factor:

$$\begin{aligned} 6x^2 - x - 12 &= 6x^2 + 8x - 9x - 12 \\ &= 2x(3x + 4) - 3(3x + 4) \\ &= (3x + 4)(2x - 3) \end{aligned}$$

Since $(3x + 4)(2x - 3) = 0$, either $3x + 4 = 0$ or $2x - 3 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} 3x + 4 &= 0 & 2x - 3 &= 0 \\ 3x &= -4 & 2x &= 3 \\ x &= -\frac{4}{3} & x &= \frac{3}{2} \end{aligned}$$

The solutions are $x = -\frac{4}{3}$ and $x = \frac{3}{2}$.

d) Simplify using the distributive property and then rearrange the equation so that the right side is 0:

$$\begin{aligned} x(x + 3) - 20 &= 5(x + 3) \\ x^2 + 3x - 20 &= 5x + 15 \\ x^2 - 2x - 35 &= 0 \end{aligned}$$

To determine this trinomial's factors, determine two numbers whose product is -35 and whose sum is -2. The numbers are -7 and 5, so the factored equation is $(x - 7)(x + 5) = 0$. It must be true that either $x - 7 = 0$ or $x + 5 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} x - 7 &= 0 & x + 5 &= 0 \\ x &= 7 & x &= -5 \end{aligned}$$

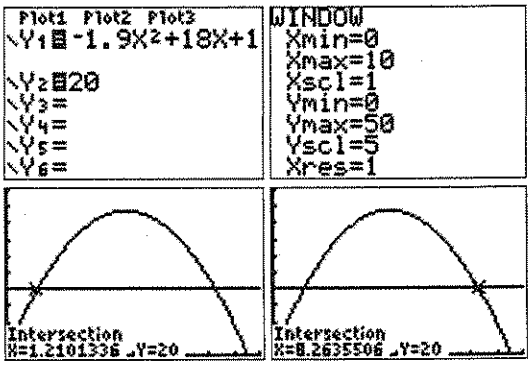
The solutions are $x = 7$ and $x = -5$.

4. The points where the bridge meets the edge of the gorge are the roots of the quadratic model. Factor the corresponding quadratic equation and solve to determine these points:

$$\begin{aligned} 0 &= 0.1x^2 - 1.2x + 2 \\ 0 &= 0.1(x^2 - 12x + 20) \\ 0 &= 0.1(x - 2)(x - 10) \end{aligned}$$

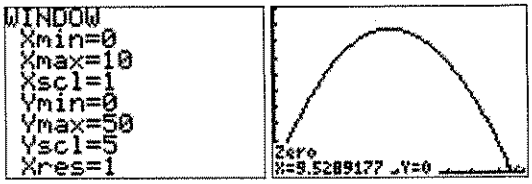
The solutions to this equation are $x = 2$ and $x = 10$. Since the bridge touches the gorge at a point 2 m from the start of the bridge and touches the other side 10 m from the start of the bridge, the bridge is $10 - 2 = 8$ m long.

5. a) Using a graphing calculator, plot both the height equation $h = -1.9t^2 + 18t + 1$ and the equation $y = 20$ to determine the points where the ball is 20 m off the ground. Use the intersect operation to locate the intersection points of the two graphs.



The ball is 20 m off the ground approximately 1.2 s and approximately 8.3 s after it was thrown.

b) The ball hits the surface when $h = 0$. Graph the height equation to determine the point at which $h = 0$:



The height is 0 when $t \doteq 9.5$, so the ball hits the surface approximately 9.5 s after it was thrown.

6. a) The coefficient of x, b, is 8. Determine c using

$$c = \left(\frac{b}{2}\right)^2:$$

$$c = \left(\frac{8}{2}\right)^2$$

$$c = 4^2$$

$$c = 16$$

The value of c that gives a perfect-square trinomial is 16.

b) The coefficient of x, b, is -10. Determine c using

$$c = \left(\frac{b}{2}\right)^2:$$

$$c = \left(-\frac{10}{2}\right)^2$$

$$c = 5^2$$

$$c = 25$$

The value of c that gives a perfect-square trinomial is 25.

c) The coefficient of x, b, is 5. Determine c using

$$c = \left(\frac{b}{2}\right)^2:$$

$$c = \left(\frac{5}{2}\right)^2$$

$$c = 2.5^2$$

$$c = 6.25$$

The value of c that gives a perfect-square trinomial is 6.25.

d) The coefficient of x, b, is -7. Determine c using

$$c = \left(\frac{b}{2}\right)^2:$$

$$c = \left(-\frac{7}{2}\right)^2$$

$$c = 3.5^2$$

$$c = 12.25$$

The value of c that gives a perfect-square trinomial is 12.25.

e) First, factor a out of the expression:

$-4x^2 + 24x = -4(x^2 - 6x)$

Determine c:

$$c = a\left(\frac{b}{2}\right)^2$$

$$c = -4\left(\frac{-6}{2}\right)^2$$

$$c = -36$$

The value of c that gives a perfect-square trinomial is -36.

f) First, factor a out of the expression:

$2x^2 - 18x = 2(x^2 - 9x)$

Determine c:

$$c = a\left(\frac{b}{2}\right)^2$$

$$c = 2\left(\frac{-9}{2}\right)^2$$

$$c = 40.5$$

The value of c that gives a perfect-square trinomial is 40.5.

7. a) The square of half of the coefficient of the x term is $\left(\frac{6}{2}\right)^2 = 9$. Add and subtract this to the relation and then factor to complete the square:

$$y = x^2 + 6x - 3$$
$$y = x^2 + 6x + 9 - 9 - 3$$
$$y = (x + 3)^2 - 12$$

b) The square of half of the coefficient of the x term is $\left(-\frac{4}{2}\right)^2 = 4$. Add and subtract this to the relation and then factor to complete the square:

$$y = x^2 - 4x + 5$$
$$y = x^2 - 4x + 4 - 4 + 5$$
$$y = (x - 2)^2 + 1$$

c) First, factor out 2 from the first two terms:

$$y = 2x^2 + 16x + 30$$
$$y = 2(x^2 + 8x) + 30$$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is $\left(\frac{8}{2}\right)^2 = 16$:

$$y = 2(x^2 + 8x + 16 - 16) + 30$$

Finish completing the square by factoring the trinomial and then simplifying:

$y = 2[(x + 4)^2 - 16] + 30$
$y = 2(x + 4)^2 - 32 + 30$
$y = 2(x + 4)^2 - 2$

d) First, factor out –3 from the first two terms:

$y = -3x^2 - 18x - 17$
$y = -3(x^2 + 6x) - 17$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is $\left(\frac{6}{2}\right)^2 = 9$:

$y = -3(x^2 + 6x + 9 - 9) - 17$

Finish completing the square by factoring the trinomial and then simplifying:

$y = -3[(x + 3)^2 - 9] - 17$
$y = -3(x + 3)^2 + 27 - 17$
$y = -3(x + 3)^2 + 10$

e) First, factor out 2 from the first two terms:

$y = 2x^2 + 10x + 8$
$y = 2(x^2 + 5x) + 8$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is $\left(\frac{5}{2}\right)^2 = 6.25$:

$y = 2(x^2 + 5x + 6.25 - 6.25) + 8$

Finish completing the square by factoring the trinomial and then simplifying:

$y = 2[(x + 2.5)^2 - 6.25] + 8$
$y = 2(x + 2.5)^2 - 12.5 + 8$
$y = 2(x + 2.5)^2 - 4.5$

f) First, factor out –3 from the first two terms:

$y = -3x^2 + 9x - 2$
$y = -3(x^2 - 3x) - 2$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is $\left(-\frac{3}{2}\right)^2 = 2.25$:

$y = -3(x^2 - 3x + 2.25 - 2.25) - 2$

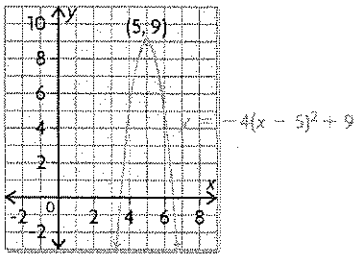
Finish completing the square by factoring the trinomial and then simplifying:

$y = -3[(x - 1.5)^2 - 2.25] - 2$
$y = -3(x - 1.5)^2 + 6.75 - 2$
$y = -3(x - 1.5)^2 + 4.75$

8. a) $y = -4x^2 + 40x - 91$
$y = -4(x^2 - 10x) - 91$
$y = -4(x^2 - 10x + 25 - 25) - 91$
$y = -4[(x - 5)^2 - 25] - 91$
$y = -4(x - 5)^2 + 100 - 91$
$y = -4(x - 5)^2 + 9$

b) From the vertex form of the relation, the vertex is (5, 9). The relation is symmetric about the x-coordinate of the vertex, therefore the equation of the axis of symmetry is $x = 5$.

c)



9. The maximum revenue and the price increase needed to earn this revenue can be obtained from the vertex of the revenue model. Complete the square to determine the vertex:

$R = -10x^2 + 100x + 3750$
$R = -10(x^2 - 10x) + 3750$
$R = -10(x^2 - 10x + 25 - 25) + 3750$
$R = -10[(x - 5)^2 - 25] + 3750$
$R = -10(x - 5)^2 + 250 + 3750$
$R = -10(x - 5)^2 + 4000$

From this form of the relation, the vertex is (5, 4000). This indicates that she will maximize her revenue if she increases the selling price by \$5/kg, which is a total price of \$20/kg.

10. The minimum height of the spider can be obtained from the vertex of the height model. Complete the square to determine the vertex:

$h = 10t^2 - 40t + 240$
$h = 10(t^2 - 4t) + 240$
$h = 10(t^2 - 4t + 4 - 4) + 240$
$h = 10[(t - 2)^2 - 4) + 240$
$h = 10(t - 2)^2 - 40 + 240$
$h = 10(t - 2)^2 + 200$

From this form of the relation, the vertex is (2, 200). This indicates that the spider will reach its minimum height from the ground, 200 cm, after 2 seconds.

Lesson 6.4: The Quadratic Formula, page 342

1. a) The equation is in the correct form.
$a = 1, b = 5, c = -2$

b) The coefficient of x in the equation is 0.
$a = 4, b = 0, c = -3$

c) The constant value, c, is 0 in the equation.
$a = 1, b = 6, c = 0$

d) Simplify the equation by multiplying using the distributive property:

$2x(x - 5) = x^2 + 1$
$2x^2 - 10x = x^2 + 1$

Rearrange the equation so the right side is 0:

$x^2 - 10x = 1$
$x^2 - 10x - 1 = 0$

Identify the values of a, b, and c:
$a = 1, b = -10, c = -1$

2. a) i) To determine the factors, determine two numbers whose sum is 18 and whose product is –63. The numbers are –3 and 21.

$x^2 + 18x - 63 = 0$
$(x - 3)(x + 21) = 0$
$x - 3 = 0$ or $x + 21 = 0$
$x = 3 \qquad x = -21$

The roots are $x = 3$ and $x = -21$.

ii) Identify the values of a, b, and c: $a = 1$, $b = 18$, $c = -63$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute the values for a, b, and c and simplify:

$$x = \frac{-(18) \pm \sqrt{(18)^2 - 4(1)(-63)}}{2(1)}$$

$$x = \frac{-18 \pm \sqrt{324 + 252}}{2}$$

$$x = \frac{-18 \pm \sqrt{576}}{2}$$

$$x = \frac{-18 \pm 24}{2}$$

The roots are $x = 3$ and $x = -21$.

iii) Answers may vary, e.g., I prefer factoring because it is faster.

b) i) Decompose the middle term by determining two numbers whose sum is -10 and whose product is $(8)(-3) = -24$. The numbers are 2 and -12.

$$8x^2 - 10x - 3 = 0$$
$$8x^2 + 2x - 12x - 3 = 0$$

Factor the first two terms and the last two terms, then divide out the common factor:

$$2x(4x + 1) - 3(4x + 1) = 0$$
$$(4x + 1)(2x - 3) = 0$$

Set each factor equal to zero and solve each equation:

$$4x + 1 = 0 \text{ or } 2x - 3 = 0$$
$$4x = -1 \qquad 2x = 3$$
$$x = -\frac{1}{4} \qquad x = \frac{3}{2}$$

The roots are $x = -\frac{1}{4}$ and $x = \frac{3}{2}$.

ii) Identify the values of a, b, and c:

$a = 8$, $b = -10$, $c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute the values for a, b, and c and simplify:

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(8)(-3)}}{2(8)}$$

$$x = \frac{10 \pm \sqrt{100 + 96}}{16}$$

$$x = \frac{10 \pm 14}{16}$$

The roots are $x = -\frac{1}{4}$ and $x = \frac{3}{2}$.

iii) Answers may vary, e.g., I prefer the quadratic formula because it is more straightforward.

3. a) Rearrange the equation so that the right side is 0:

$$2x^2 = 50$$
$$x^2 = 25$$
$$\sqrt{x^2} = \pm\sqrt{25}$$
$$x = \pm 5$$

The roots are $x = 5$ and $x = -5$.

b) Isolate x^2 and then take the square root of both sides:

$$x^2 - 1 = 0$$
$$x^2 = 1$$
$$\sqrt{x^2} = \pm\sqrt{1}$$
$$x = \pm 1$$

The roots are $x = 1$ and $x = -1$.

c) Isolate x^2 and then take the square root of both sides:

$$3x^2 - 2 = 10$$
$$3x^2 = 12$$
$$x^2 = 4$$
$$\sqrt{x^2} = \pm\sqrt{4}$$
$$x = \pm 2$$

The roots are $x = 2$ and $x = -2$.

d) First, simplify the equation by multiplying using the distributive property:

$$x(x - 2) = 36 - 2x$$
$$x^2 - 2x = 36 - 2x$$

Notice that the $-2x$ terms on both sides cancel out. Take the square root of both sides:

$$x^2 = 36$$
$$\sqrt{x^2} = \pm\sqrt{36}$$
$$x = \pm 6$$

The roots are $x = 6$ and $x = -6$.

4. a) Isolate $(x + 1)^2$ and then take the square root of both sides:

$$(x + 1)^2 - 16 = 0$$
$$(x + 1)^2 = 16$$
$$\sqrt{(x+1)^2} = \pm\sqrt{16}$$
$$x + 1 = \pm 4$$

Now solve for x:

$$x = -1 \pm 4$$

The roots are $x = -5$ and $x = 3$.

b) Isolate $(x + 5)^2$ and then take the square root of both sides:

$$-2(x + 5)^2 + 2 = 0$$
$$-2(x + 5)^2 = -2$$
$$(x + 5)^2 = 1$$
$$\sqrt{(x+5)^2} = \pm\sqrt{1}$$
$$x + 5 = \pm 1$$

Now solve for x:

$$x = -5 \pm 1$$

The roots are $x = -4$ and $x = -6$.

c) Isolate $(x - 7)^2$ and then take the square root of both sides:

$$-3(x - 7)^2 + 3 = 0$$
$$-3(x - 7)^2 = -3$$
$$(x - 7)^2 = 1$$
$$\sqrt{(x-7)^2} = \pm\sqrt{1}$$
$$x - 7 = \pm 1$$

Now solve for x:

$$x = 7 \pm 1$$

The roots are $x = 6$ and $x = 8$.

d) Isolate $(x-2)^2$ and then take the square root of both sides:

$$4(x-2)^2 - 5 = 0$$
$$4(x-2)^2 = 5$$
$$(x-2)^2 = \frac{5}{4}$$
$$\sqrt{(x-2)^2} = \pm\sqrt{\frac{5}{4}}$$
$$x-2 = \pm\sqrt{\frac{5}{4}}$$

Now solve for x:

$$x = 2 \pm \sqrt{\frac{5}{4}}$$

The roots are $x \doteq 3.12$ and $x \doteq 0.88$.

e) Isolate $(x+3)^2$ and then take the square root of both sides:

$$-6(x+3)^2 + 12 = 0$$
$$-6(x+3)^2 = -12$$
$$(x+3)^2 = 2$$
$$\sqrt{(x+3)^2} = \pm\sqrt{2}$$
$$x+3 = \pm\sqrt{2}$$

Now solve for x:

$$x = -3 \pm \sqrt{2}$$

The roots are $x \doteq -1.59$ and $x \doteq -4.41$.

f) Isolate $(x-4)^2$ and then take the square root of both sides:

$$0.25(x-4)^2 - 4 = 0$$
$$0.25(x-4)^2 = 4$$
$$(x-4)^2 = 16$$
$$\sqrt{(x-4)^2} = \pm\sqrt{16}$$
$$x-4 = \pm 4$$

Now solve for x:

$$x = 4 \pm 4$$

The roots are $x = 0$ and $x = 8$.

5. a) Identify the values of a, b, and c:
$a = 6$, $b = -1$, $c = -15$
Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(6)(-15)}}{2(6)}$$
$$x = \frac{1 \pm \sqrt{1+360}}{12}$$
$$x = \frac{1 \pm \sqrt{361}}{12}$$
$$x = \frac{1 \pm 19}{12}$$

The roots are $x = -1.5$ and $x \doteq 1.67$.

b) Identify the values of a, b, and c: $a = 4$, $b = -20$, $c = 25$
Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(4)(25)}}{2(4)}$$
$$x = \frac{20 \pm \sqrt{400-400}}{8}$$
$$x = \frac{20 \pm \sqrt{0}}{8}$$
$$x = \frac{20}{8}$$

The root is $x = 2.5$.

c) Identify the values of a, b, and c: $a = 1$, $b = 0$, $c = -16$
Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-0 \pm \sqrt{0^2 - 4(1)(-16)}}{2(1)}$$
$$x = \frac{\pm\sqrt{64}}{2}$$
$$x = \pm\frac{8}{2}$$

The roots are $x = 4$ and $x = -4$.

d) Identify the values of a, b, and c: $a = 5$, $b = -11$, $c = 0$
Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(5)(0)}}{2(5)}$$
$$x = \frac{11 \pm \sqrt{121}}{10}$$
$$x = \frac{11 \pm 11}{10}$$

The roots are $x = 0$ and $x = 2.2$.

e) Identify the values of a, b, and c: $a = 1$, $b = 9$, $c = 20$
Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-9 \pm \sqrt{9^2 - 4(1)(20)}}{2(1)}$$
$$x = \frac{-9 \pm \sqrt{81-80}}{2}$$
$$x = \frac{-9 \pm \sqrt{1}}{2}$$
$$x = \frac{-8}{2} \text{ or } x = \frac{-10}{2}$$

The roots are $x = -4$ and $x = -5$.

f) Rearrange the equation so that the right side is 0:

$$12x^2 - 40 = 17x$$
$$12x^2 - 17x - 40 = 0$$

Identify the values of a, b, and c:
$a = 12$, $b = -17$, $c = -40$
Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-17) \pm \sqrt{(-17)^2 - 4(12)(-40)}}{2(12)}$$
$$x = \frac{17 \pm \sqrt{289 + 1920}}{24}$$
$$x = \frac{17 \pm \sqrt{2209}}{24}$$
$$x = \frac{17 \pm 47}{24}$$

The roots are $x = -1.25$ and $x \doteq 2.67$.

6. Answers may vary, e.g., yes, since all the answers in question 5 are integers or fractions, the equations could have been solved by factoring.

7. Answers may vary, e.g., they will not have square roots. Since the solutions are the same whether you use factoring or the quadratic formula, and the solutions determined from factoring contain no square roots, the solutions found using the quadratic formula cannot contain roots either.

8. a) Identify the values of a, b, and c: $a = 1$, $b = -4$, $c = -1$
Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2(1)}$$
$$x = \frac{4 \pm \sqrt{16 + 4}}{2}$$
$$x = \frac{4 \pm \sqrt{20}}{2}$$

The roots are $x \doteq 4.24$ and $x \doteq -0.24$.

b) Identify the values of a, b, and c: $a = 5$, $b = -6$, $c = -2$
Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(5)(-2)}}{2(5)}$$
$$x = \frac{6 \pm \sqrt{36 + 40}}{10}$$
$$x = \frac{6 \pm \sqrt{76}}{10}$$

The roots are $x \doteq -0.27$ and $x \doteq 1.47$.

c) Identify the values of a, b, and c: $a = 3$, $b = 8$, $c = 2$
Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(3)(2)}}{2(3)}$$
$$x = \frac{-8 \pm \sqrt{64 - 24}}{6}$$
$$x = \frac{-8 \pm \sqrt{40}}{6}$$

The roots are $x \doteq -0.28$ and $x \doteq -2.39$.

d) Identify the values of a, b, and c: $a = 2$, $b = -1$, $c = -3$
Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-3)}}{2(2)}$$
$$x = \frac{1 \pm \sqrt{1 + 24}}{4}$$
$$x = \frac{1 \pm \sqrt{25}}{4}$$
$$x = \frac{1 \pm 5}{4}$$

The roots are $x = -1$ and $x = 1.5$.

e) Identify the values of a, b, and c: $a = 1$, $b = -5$, $c = 3$
Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(3)}}{2(1)}$$
$$x = \frac{5 \pm \sqrt{25 - 12}}{2}$$
$$x = \frac{5 \pm \sqrt{13}}{2}$$

The roots are $x \doteq 0.70$ and $x \doteq 4.30$.

f) Identify the values of a, b, and c: $a = -3$, $b = 12$, $c = -7$
Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-12 \pm \sqrt{12^2 - 4(-3)(-7)}}{2(-3)}$$
$$x = \frac{-12 \pm \sqrt{144 - 84}}{-6}$$
$$x = \frac{-12 \pm \sqrt{60}}{-6}$$

The roots are $x \doteq 3.29$ and $x \doteq 0.71$.

9. a) Simplify the equation by multiplying, using the distributive property:

$$2x^2 - 5x = 3(x + 4)$$
$$2x^2 - 5x = 3x + 12$$

Rearrange the equation so that the right side is 0:

$$2x^2 - 8x - 12 = 0$$

Identify the values of a, b, and c:

$a = 2$, $b = -8$, $c = -12$

Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(-12)}}{2(2)}$$
$$x = \frac{8 \pm \sqrt{64 + 96}}{4}$$
$$x = \frac{8 \pm \sqrt{160}}{4}$$

The roots are $x \doteq -1.16$ and $x \doteq 5.16$.

b) Simplify the equation by multiplying, using the distributive property:

$$(x + 4)^2 = 2(x + 5)$$
$$x^2 + 8x + 16 = 2(x + 5)$$
$$x^2 + 8x + 16 = 2x + 10$$

Rearrange the equation so that the right side is 0:

$$x^2 + 6x + 6 = 0$$

Identify the values of a, b, and c:

$a = 1$, $b = 6$, $c = 6$

Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(6)}}{2(1)}$$
$$x = \frac{-6 \pm \sqrt{36 - 24}}{2}$$
$$x = \frac{-6 \pm \sqrt{12}}{2}$$

The roots are $x \doteq -1.27$ and $x \doteq -4.73$.

c) Simplify the equation by multiplying, using the distributive property:

$$x(x + 3) = 5 - x^2$$
$$x^2 + 3x = 5 - x^2$$

Rearrange the equation so that the right side is 0:

$$2x^2 + 3x - 5 = 0$$

Identify the values of a, b, and c:

$a = 2$, $b = 3$, $c = -5$

Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-5)}}{2(2)}$$
$$x = \frac{-3 \pm \sqrt{9 + 40}}{4}$$
$$x = \frac{-3 \pm \sqrt{49}}{4}$$
$$x = \frac{-3 \pm 7}{4}$$

The roots are $x = -2.5$ and $x = 1$.

d) Simplify the equation by multiplying, using the distributive property:

$$3x(x + 4) = (4x - 1)^2$$
$$3x^2 + 12x = (4x - 1)^2$$
$$3x^2 + 12x = 16x^2 - 8x + 1$$

Rearrange the equation so that the left side is 0:

$$0 = 13x^2 - 20x + 1$$

Identify the values of a, b, and c:

$a = 13$, $b = -20$, $c = 1$

Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(13)(1)}}{2(13)}$$
$$x = \frac{20 \pm \sqrt{400 - 52}}{26}$$
$$x = \frac{20 \pm \sqrt{348}}{26}$$

The roots are $x \doteq 1.49$ and $x \doteq 0.05$.

e) Simplify the equation by multiplying, using the distributive property:

$$(x - 2)(2x + 3) = x + 1$$
$$2x^2 - x - 6 = x + 1$$

Rearrange the equation so that the right side is 0:

$$2x^2 - 2x - 7 = 0$$

Identify the values of a, b, and c:

$a = 2$, $b = -2$, $c = -7$

Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-7)}}{2(2)}$$
$$x = \frac{2 \pm \sqrt{4 + 56}}{4}$$
$$x = \frac{2 \pm \sqrt{60}}{4}$$

The roots are $x \doteq -1.44$ and $x \doteq 2.44$.

f) Simplify the equation by multiplying, using the distributive property:

$$(x-3)^2+5 = 3(x+1)$$
$$x^2-6x+14 = 3(x+1)$$
$$x^2-6x+14 = 3x+3$$

Rearrange the equation so that the right side is 0:

$$x^2-9x+11 = 0$$

Identify the values of a, b, and c:
$a = 1$, $b = -9$, $c = 11$
Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$x = \frac{-(-9) \pm \sqrt{(-9)^2-4(1)(11)}}{2(1)}$$
$$x = \frac{9 \pm \sqrt{81-44}}{2}$$
$$x = \frac{9 \pm \sqrt{37}}{2}$$

The roots are $x \doteq 1.46$ and $x \doteq 7.54$.

10. a) Identify the values of a, b, and c:
$a = 2$, $b = 5$, $c = -14$.
Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$x = \frac{-5 \pm \sqrt{5^2-4(2)(-14)}}{2(2)}$$
$$x = \frac{-5 \pm \sqrt{25+112}}{4}$$
$$x = \frac{-5 \pm \sqrt{137}}{4}$$

The roots are $x \doteq 1.68$ and $x \doteq -4.18$.

b) Rearrange the equation so that the right side is 0:

$$3x^2+7.5x = 21$$
$$3x^2+7.5x-21 = 0$$

Identify the values of a, b, and c:
$a = 3$, $b = 7.5$, $c = -21$.
Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$x = \frac{-7.5 \pm \sqrt{7.5^2-4(3)(-21)}}{2(3)}$$
$$x = \frac{-7.5 \pm \sqrt{56.25+252}}{6}$$
$$x = \frac{-7.5 \pm \sqrt{308.25}}{6}$$

The roots are $x \doteq 1.68$ and $x \doteq -4.18$.

c) Simplify the equation by multiplying, using the distributive property:

$$3x(0.4x+1) = 8.4$$
$$1.2x^2+3x = 8.4$$

Rearrange the equation so that the right side is 0:

$$1.2x^2+3x-8.4 = 0$$

Identify the values of a, b, and c:
$a = 1.2$, $b = 3$, $c = -8.4$
Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$x = \frac{-3 \pm \sqrt{3^2-4(1.2)(-8.4)}}{2(1.2)}$$
$$x = \frac{-3 \pm \sqrt{9+40.32}}{2.4}$$
$$x = \frac{-3 \pm \sqrt{49.32}}{2.4}$$

The roots are $x \doteq 1.68$ and $x \doteq -4.18$.

d) Rearrange the equation so that the right side is 0:

$$0.2x^2 = -0.5x+1.4$$
$$0.2x^2+0.5x-1.4 = 0$$

Identify the values of a, b, and c:
$a = 0.2$, $b = 0.5$, $c = -1.4$
Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$x = \frac{-0.5 \pm \sqrt{0.5^2-4(0.2)(-1.4)}}{2(0.2)}$$
$$x = \frac{-0.5 \pm \sqrt{0.25+1.12}}{0.4}$$
$$x = \frac{-0.5 \pm \sqrt{1.37}}{0.4}$$

The roots are $x \doteq 1.68$ and $x \doteq -4.18$.

11. a) All the solutions are the same.

b) Answers may vary, e.g., all the equations are constant multiples of each other. The equations are proportional to each other.

12. At the points of intersection the y-values of the parabolas are equal. Set the expressions for the parabolas equal to each other and rearrange the resulting equation so that the right side is 0:

$$2x^2+5x-8 = -3x^2+8x-1$$
$$5x^2-3x-7 = 0$$

Identify the values of a, b, c:
$a = 5$, $b = -3$, $c = -7$
Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$x = \frac{-(-3) \pm \sqrt{(-3)^2-4(5)(-7)}}{2(5)}$$
$$x = \frac{3 \pm \sqrt{9+140}}{10}$$
$$x = \frac{3 \pm \sqrt{149}}{10}$$

The x-values of the points of intersection are $x \doteq -0.9207$ and $x \doteq 1.5207$. Substitute -0.9207 for x in one of the equations of the parabolas to determine its corresponding y-value:

$$\begin{aligned} y &= 2x^2 + 5x - 8 \\ y &= 2(-0.9207)^2 + 5(-0.9207) - 8 \\ y &= 2(0.848) + 5(-0.9207) - 8 \\ y &= 1.696 - 4.604 - 8 \\ y &= -10.91 \end{aligned}$$

One point of intersection is about $(-0.92, -10.91)$.
Perform another substitution using 1.5207 for x:

$$\begin{aligned} y &= 2x^2 + 5x - 8 \\ y &= 2(1.5207)^2 + 5(1.5207) - 8 \\ y &= 2(2.313) + 5(1.5207) - 8 \\ y &= 4.626 + 7.604 - 8 \\ y &= 4.23 \end{aligned}$$

The other point of intersection is about $(1.52, 4.23)$.

13. Since the triangle contains a right angle, its sides are related by the Pythagorean theorem. We have that:
$c^2 = a^2 + b^2$
Substitute the side lengths x, $x + 7$, and $x + 9$ into the equation, and then simplify:

$$\begin{aligned} (x + 9)^2 &= x^2 + (x + 7)^2 \\ x^2 + 18x + 81 &= x^2 + x^2 + 14x + 49 \\ x^2 + 18x + 81 &= 2x^2 + 14x + 49 \end{aligned}$$

Rearrange the equation so that the left side is 0:

$$0 = x^2 - 4x - 32$$

This trinomial is factorable over the set of integers. To determine the factors, determine two numbers whose product is -32 and whose sum is -4. The numbers are -8 and 4.

$$\begin{aligned} 0 &= x^2 - 4x - 32 \\ 0 &= (x - 8)(x + 4) \end{aligned}$$

Set each factor to 0 and solve each equation:

$$\begin{aligned} x - 8 = 0 \text{ or } x + 4 &= 0 \\ x = 8 \qquad\quad x &= -4 \end{aligned}$$

The roots are $x = 8$ and $x = -4$, but the side lengths of the triangle must all be positive, therefore the only valid solution is $x = 8$.

14. a) The times when the stunt diver is exactly 15 m high can be found by substituting 15 for h in the model:

$$\begin{aligned} h &= -4.9t^2 + 1.2t + 15 \\ 15 &= -4.9t^2 + 1.2t + 15 \end{aligned}$$

Rearrange the equation so that the left side is 0:

$$0 = -4.9t^2 + 1.2t$$

Factor out t from the two remaining terms:

$$0 = t(-4.9t + 1.2)$$

Set each factor equal to zero and solve each equation:

$$\begin{aligned} t = 0 \text{ or } -4.9t + 1.2 &= 0 \\ -4.9x &= -1.2 \\ x &= \frac{-1.2}{-4.9} \end{aligned}$$

The times when the diver is exactly 15 m high are $t = 0$ and $t \doteq 0.2$. The diver is 15 m high at the beginning of the dive and is 15 m high again about 0.2 s later, so the total time the diver is above 15 m is about 0.2 s.

b) The time when the stunt diver hits the water can be found by substituting 0 for h in the model:

$$\begin{aligned} h &= -4.9t^2 + 1.2t + 15 \\ 0 &= -4.9t^2 + 1.2t + 15 \end{aligned}$$

This trinomial does not appear to be factorable over the set of integers. Identify the values of a, b, and c to solve the quadratic equation using the quadratic formula:
$a = -4.9$, $b = 1.2$, $c = 15$
Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-1.2 \pm \sqrt{1.2^2 - 4(-4.9)(15)}}{2(-4.9)}$$

$$t = \frac{-1.2 \pm \sqrt{1.44 + 294}}{-9.8}$$

$$t = \frac{-1.2 \pm \sqrt{295.44}}{-9.8}$$

The roots of the quadratic equation are $t \doteq -1.6$ and $t \doteq 1.9$. The time the stunt diver is in the air must be positive, so the root $t \doteq -1.6$ can be ignored. The root $t \doteq 1.9$ indicates that the stunt diver hit the water after about 1.9 s, so the stunt diver was in the air for about 1.9 s.

15. Let x represent the rectangle's width. Now, $x + 5$ represents the rectangle's length, and it is given that its diagonal is 18. The sides of the rectangle are related to its diagonal by the Pythagorean theorem. We have that:
$c^2 = a^2 + b^2$
Substitute the side lengths x, $x + 5$, and 18 into the equation, and then simplify:

$$\begin{aligned} 18^2 &= x^2 + (x + 5)^2 \\ 324 &= x^2 + x^2 + 10x + 25 \\ 324 &= 2x^2 + 10x + 25 \end{aligned}$$

Rearrange the equation so that the left side is 0:

$$0 = 2x^2 + 10x - 299$$

This trinomial does not appear to be factorable over the set of integers. Identify the values of a, b, and c to solve the quadratic equation using the quadratic formula:
$a = 2$, $b = 10$, $c = -299$
Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(10) \pm \sqrt{(10)^2 - 4(2)(-299)}}{2(2)}$$

$$x = \frac{-10 \pm \sqrt{100 + 2392}}{4}$$

$$x = \frac{-10 \pm \sqrt{2492}}{4}$$

The roots of the quadratic equation are $x \doteq -14.98$ and $x \doteq 9.98$. The width of the rectangle must be positive, so the root $x \doteq -14.98$ can be ignored. The root $x \doteq 9.98$ indicates that the width of the rectangle is about 9.98 cm. The length of the rectangle is 5 cm longer than the width, or 14.98 cm. The dimensions of the rectangle are about 9.98 cm by about 14.98 cm.

16. Let x represent the side length of the lawn. The area of the lawn is then $(x)(x) = x^2$. The area of the walkway is: area of entire square – area of lawn $= (x + 4)^2 - x^2$

Set these two areas equal to each other and simplify:

$x^2 = (x+4)^2 - x^2$

$x^2 = x^2 + 8x + 16 - x^2$

$x^2 = 8x + 16$

Rearrange the equation so that the right side is 0:

$x^2 - 8x - 16 = 0$

This trinomial does not appear to be factorable over the set of integers. Identify the values of a, b, and c to solve the quadratic equation using the quadratic formula:

$a = 1, b = -8, c = -16$

Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-16)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{64 + 64}}{2}$$

$$x = \frac{8 \pm \sqrt{128}}{2}$$

The roots of the quadratic equation are $x \doteq -1.7$ and $x \doteq 9.7$. The side length of the lawn must be positive, so the root $x \doteq -1.7$ can be ignored. The root $x \doteq 9.7$ indicates that the side length of the lawn is about 9.7 m. The dimensions of the lawn are about 9.7 m by about 9.7 m.

17. Answers may vary, e.g.,

Strategy	Advantages	Example
factoring	-is usually quick if the equation is easily factorable -allows roots to be seen from the factors -involves less complicated calculations	$x^2 + 5x - 4 = 0$
quadratic formula	-can be used when the equation is not factorable over integers -can be used when coefficients are great	$2x^2 + 5x - 12 = 0$ $3.5x^2 + 15.7x + 2.8 = 0$ $105x^2 - 187x - 156 = 0$

18. Substitute the equation of the line, $y = 2x + 5$, into the equation for the circle and simplify:

$x^2 + y^2 = 36$

$x^2 + (2x + 5)^2 = 36$

$x^2 + 4x^2 + 20x + 25 = 36$

$5x^2 + 20x + 25 = 36$

Rearrange the equation so that the right side is 0:

$5x^2 + 20x - 11 = 0$

This trinomial does not appear to be factorable over the set of integers. Identify the values of a, b, and c to solve the quadratic equation using the quadratic formula:

$a = 5, b = 20, c = -11$

Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-20 \pm \sqrt{20^2 - 4(5)(-11)}}{2(5)}$$

$$x = \frac{-20 \pm \sqrt{400 + 220}}{10}$$

$$x = \frac{-20 \pm \sqrt{620}}{10}$$

The roots are $x \doteq 0.49$ and $x \doteq -4.49$, therefore the x-values of the points of intersection are about 0.49 and about –4.49. Substitute 0.49 for x in the equation of the line to determine its corresponding y-value:

$y = 2x + 5$

$y = 2(0.49) + 5$

$y = 0.98 + 5$

$y = 5.98$

One point of intersection is about (0.49, 5.98).

Perform another substitution using –4.49 for x:

$y = 2x + 5$

$y = 2(-4.49) + 5$

$y = -8.98 + 5$

$y = -3.98$

The second point of intersection is about (–4.49, –3.98).

19. Answers may vary, e.g.,

a) First, rearrange the equations for each root so that the right side is 0:

$x = -3$ or $x = 5$

$x + 3 = 0 \quad x - 5 = 0$

Then, multiply the two expressions for the roots and set the product equal to 0:

$0 = (x + 3)(x - 5)$

Expand and simplify to obtain a quadratic equation in standard form:

$0 = x^2 - 5x + 3x - 15$

$0 = x^2 - 2x - 15$

b) First, rearrange the equations for each root so that the right side is 0:

$$x = \frac{2 + \sqrt{5}}{3} \quad \text{or} \quad x = \frac{2 - \sqrt{5}}{3}$$

$$x - \frac{2 + \sqrt{5}}{3} = 0 \qquad x - \frac{2 - \sqrt{5}}{3} = 0$$

Then, multiply the two expressions for the roots and set the product equal to 0:

$$0 = \left(x - \frac{2 + \sqrt{5}}{3}\right)\left(x - \frac{2 - \sqrt{5}}{3}\right)$$

Expand and simplify:

$$0 = x^2 - \frac{2+\sqrt{5}}{3}x - \frac{2-\sqrt{5}}{3}x + \left(\frac{2+\sqrt{5}}{3}\right)\left(\frac{2-\sqrt{5}}{3}\right)$$

$$0 = x^2 - \frac{4}{3}x + \frac{(2+\sqrt{5})(2-\sqrt{5})}{9}$$

$$0 = x^2 - \frac{4}{3}x + \frac{4-5}{9}$$

$$0 = x^2 - \frac{4}{3}x - \frac{1}{9}$$

Multiply by 9 to eliminate the denominators of the coefficients and obtain a quadratic equation in standard form:

$$0 = 9\left(x^2 - \frac{4}{3}x - \frac{1}{9}\right)$$

$$0 = 9x^2 - 12x - 1$$

20. Let x represent the side length of the smallest side. Now, the other two sides have lengths $x + 2$ and $x + 4$. Because it is a right triangle, the three sides are related by the Pythagorean theorem: $c^2 = a^2 + b^2$
Substitute the side lengths of the triangle into the equation and simplify:

$$(x+4)^2 = (x+2)^2 + x^2$$
$$x^2 + 8x + 16 = x^2 + 4x + 4 + x^2$$
$$x^2 + 8x + 16 = 2x^2 + 4x + 4$$

Rearrange the equation so that the left side is 0:

$$0 = x^2 - 4x - 12$$

To determine the factors, determine two numbers whose product is –12 and whose sum is –4. The numbers are –6 and 2.

$$0 = x^2 - 4x - 12$$
$$0 = (x-6)(x+2)$$

Set each factor to 0 and solve each equation:

$$x - 6 = 0 \text{ or } x + 2 = 0$$
$$x = 6 \qquad x = -2$$

The roots are $x = 6$ and $x = -2$, but the side lengths of the triangle must all be positive, therefore the only valid solution is $x = 6$. Therefore, the smallest side of the triangle is 6 cm and the other two sides have lengths (6) + 2 = 8 and (6) + 4 = 10. The dimensions of the triangle are 6 cm, 8 cm, and 10 cm.

Lesson 6.5: Interpreting Quadratic Equation Roots, page 349

1. a) Using the quadratic formula: Identify the values of a, b, and c: $a = 1$, $b = -6$, $c = 5$. Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - 20}}{2}$$

$$x = \frac{6 \pm \sqrt{16}}{2}$$

$$x = \frac{6 \pm 4}{2}$$

The roots are $x = 1$ and $x = 5$.

Factoring: To determine this trinomial's factors, determine two numbers whose product is 5 and whose sum is –6. The numbers are –1 and –5, so the factored equation is $(x - 1)(x - 5) = 0$. It must be true that either $x - 1 = 0$ or $x - 5 = 0$. Solve the two equations to obtain both solutions:

$$x - 1 = 0 \qquad x - 5 = 0$$
$$x = 1 \qquad x = 5$$

The roots are $x = 1$ and $x = 5$.
b) The graph has two x-intercepts.
c) Identify the values of a, b, and c:
$a = 1$, $b = -6$, $c = 5$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$$D = b^2 - 4ac$$
$$D = (-6)^2 - 4(1)(5)$$
$$D = 36 - 20$$
$$D = 16$$

Since $D > 0$, there are two distinct real roots, so the answer to part b) is correct.
2. a) The vertex of the corresponding quadratic relation is (1, 3), and the graph opens upward since $a > 0$. The parabola never passes through the x-axis so there are no real solutions. This indicates that $D < 0$.
b) The vertex of the corresponding quadratic relation is (5, 8), and the graph opens downward since $a < 0$. The parabola passes through the x-axis in two places so there are two real solutions. This indicates that $D > 0$.
c) The vertex of the corresponding quadratic relation is (–3, 0). Since the vertex is on the x-axis there is one real solution. This indicates that $D = 0$.
3. a) Identify the values of a, b, and c:
$a = 1$, $b = 3$, $c = -5$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$$D = b^2 - 4ac$$
$$D = 3^2 - 4(1)(-5)$$
$$D = 9 + 20$$
$$D = 29$$

Since $D > 0$, there are two real solutions.
b) Identify the values of a, b, and c:
$a = 6$, $b = 5$, $c = 12$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$$D = b^2 - 4ac$$
$$D = 5^2 - 4(6)(12)$$
$$D = 25 - 288$$
$$D = -263$$

Since $D < 0$, there are no real solutions.
c) Rearrange the equation so that the right side is 0:

$$-x^2 + 8x = 12$$
$$-x^2 + 8x - 12 = 0$$

Identify the values of a, b, and c:
$a = -1$, $b = 8$, $c = -12$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$$D = b^2 - 4ac$$
$$D = 8^2 - 4(-1)(-12)$$
$$D = 64 - 48$$
$$D = 16$$

Since $D > 0$, there are two real solutions.

d) Identify the values of a, b, and c:
$a = -2$, $b = 8$, $c = -8$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$$D = b^2 - 4ac$$
$$D = 8^2 - 4(-2)(-8)$$
$$D = 64 - 64$$
$$D = 0$$

Since $D = 0$, there is one real solution.
e) Rearrange the equation so that the right side is 0:

$$3x^2 + 2x = 5x + 12$$
$$3x^2 - 3x - 12 = 0$$

Identify the values of a, b, and c:
$a = 3$, $b = -3$, $c = -12$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$$D = b^2 - 4ac$$
$$D = (-3)^2 - 4(3)(-12)$$
$$D = 9 + 144$$
$$D = 153$$

Since $D > 0$, there are two real solutions.
f) Rearrange the equation so that the left side is 0:

$$-17x - 9 = 4x^2 - 5x$$
$$0 = 4x^2 + 12x + 9$$

Identify the values of a, b, and c:
$a = 4$, $b = 12$, $c = 9$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$$D = b^2 - 4ac$$
$$D = 12^2 - 4(4)(9)$$
$$D = 144 - 144$$
$$D = 0$$

Since $D = 0$, there is one real solution.
4. a) The vertex of the relation is $(-2, -5)$, and the parabola opens upward since $a > 0$. Since the vertex is below the x-axis and the parabola opens upward, it passes through the x-axis in two places. This indicates $D > 0$.
b) The vertex of the relation is $(-5, -8)$, and the parabola opens downward since $a < 0$. Since the vertex is below the x-axis and the parabola opens downward, it never passes through the x-axis. This indicates that $D < 0$.
c) The vertex of the relation is $(7, 0)$. Since the vertex is on the x-axis it passes through the x-axis once. This indicates that $D = 0$.
d) The vertex of the relation is $(12, 81)$, and the parabola opens upward since $a > 0$. Since the vertex is above the x-axis and the parabola opens upward, it never passes through the x-axis. This indicates that $D < 0$.
e) The vertex of the relation is $(0, 5)$, and the parabola opens downward since $a < 0$. Since the vertex is above the x-axis and the parabola opens downward, it passes through the x-axis in two places. This indicates that $D > 0$.
f) The vertex of the relation is $(0, 0)$. Since the vertex is on the x-axis it passes through the x-axis once. This indicates that $D = 0$.
5. a) The vertex of the relation is $(-2, 5)$ and the parabola opens upward since $a > 0$. Since the vertex is above the x-axis and the parabola opens upward, it never passes through the x-axis so there are no zeros. This indicates that $D < 0$.
b) Identify the values of a, b, and c:
$a = -2$, $b = 3$, $c = -7$.
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$$D = b^2 - 4ac$$
$$D = 3^2 - 4(-2)(-7)$$
$$D = 9 - 56$$
$$D = -47$$

Since $D < 0$, there are no zeros.
c) First expand the right side of the equation:

$$y = x(x - 7)$$
$$y = x^2 - 7x$$

Identify the values of a, b, and c:
$a = 1$, $b = -7$, $c = 0$.
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$$D = b^2 - 4ac$$
$$D = (-7)^2 - 4(1)(0)$$
$$D = 49 - 0$$
$$D = 49$$

Since $D > 0$, there are two zeros.
d) The relation is equivalent to $y = 5(x + 2)^2$. The vertex of the relation is $(-2, 0)$. Since the vertex is on the x-axis it passes through the x-axis once so there is one zero. This indicates that $D = 0$.
e) Identify the values of a, b, and c:
$a = 3$, $b = 6$, $c = -8$.
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$$D = b^2 - 4ac$$
$$D = 3^2 - 4(3)(-8)$$
$$D = 9 + 96$$
$$D = 105$$

Since $D > 0$, there are two zeros.
f) The vertex of the relation is $(0, 9)$ and the parabola opens downward since $a < 0$. Since the vertex is above the x-axis and the parabola opens downward, it passes through the x-axis in two places so there are two zeros. This indicates that $D > 0$.
6. Since she would like to earn \$500, let $r = 500$.

$$r = 250 + 5n - 2n^2$$
$$500 = 250 + 5n - 2n^2$$

If there is an amount to increase that price that produces revenue of \$500, then this equation will have at least one real solution.
Determine the number of solutions by using the discriminant. Rearrange the equation so that the left side is 0:

$$0 = -250 + 5n - 2n^2$$

Identify the values of a, b, and c:
$a = -2$, $b = 5$, $c = -250$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$$D = b^2 - 4ac$$
$$D = 5^2 - 4(-2)(-250)$$
$$D = 25 - 2000$$
$$D = -1975$$

Since $D < 0$, there are no real solutions to the equation and it is not possible for her to earn revenue of \$500.

7. a) Answers may vary, e.g., none, because the x^2 term is always positive so the height is always positive. This makes sense because the lowest point of the bridge (the vertex) should be above the water level.

b) Determine the lowest point of the bridge by determining the vertex of the model of the height. The model is already in vertex form and the vertex is (0, 24). Since the lowest point of the bridge is 24 m above the riverbed, the water level could rise 24 m before the bridge is no longer safe to use.

8. a) The relation should have one zero, e.g., the ball starts above the ground and falls downward.

b) Determine the zeros of the relation $h = -4.9t^2 + 10.78t + 1.071$ using the quadratic formula.

Identify the values of a, b, and c:

$a = -4.9$, $b = 10.78$, $c = 1.071$

Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-10.78 \pm \sqrt{10.78^2 - 4(-4.9)(1.071)}}{2(-4.9)}$$

$$t = \frac{-10.78 \pm \sqrt{116.2084 + 20.9916}}{-9.8}$$

$$t = \frac{-10.78 \pm \sqrt{137.2}}{-9.8}$$

The zeros are $x \doteq -0.10$ and $x \doteq 2.30$. The first zero is ignored since time must be positive, so there is only one valid zero.

c) Answers may vary, e.g., 5 m: twice; 7 m: once; 9 m: zero

d) For a height of 5 m, substitute $h = 5$ m into the height equation. Rearrange the equation so that the left side is 0 and then determine the discriminant:

$h = -4.9t^2 + 10.78t + 1.071$
$5 = -4.9t^2 + 10.78t + 1.071$
$0 = -4.9t^2 + 10.78t - 3.929$

Identify the values of a, b, and c:

$a = -4.9$, $b = 10.78$, $c = -3.929$.

Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$D = b^2 - 4ac$
$D = 10.78^2 - 4(-4.9)(-3.929)$
$D = 116.2084 - 77.0084$
$D = 39.2$

Since $D > 0$, there are two zeros.

Use the same method for a height of 7 m.

$h = -4.9t^2 + 10.78t + 1.071$
$7 = -4.9t^2 + 10.78t + 1.071$
$0 = -4.9t^2 + 10.78t - 5.929$

Identify the values of a, b, and c:

$a = -4.9$, $b = 10.78$, $c = -5.929$.

Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$D = b^2 - 4ac$
$D = 10.78^2 - 4(-4.9)(-5.929)$
$D = 116.2084 - 116.2084$
$D = 0$

Since $D = 0$, there is one zero.

Use the same method for a height of 9 m.

$h = -4.9t^2 + 10.78t + 1.071$
$9 = -4.9t^2 + 10.78t + 1.071$
$0 = -4.9t^2 + 10.78t - 7.929$

Identify the values of a, b, and c:

$a = -4.9$, $b = 10.78$, $c = -7.929$.

Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$D = b^2 - 4ac$
$D = 10.78^2 - 4(-4.9)(-7.929)$
$D = 116.2084 - 155.4084$
$D = -39.2$

Since $D < 0$, there are no zeros.

9. a) Identify the values of a, b, and c:

$a = 2$, $b = -4$, $c = 1.5$

Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$D = b^2 - 4ac$
$D = (-4)^2 - 4(2)(1.5)$
$D = 16 - 12$
$D = 4$

Since $D > 0$, there are two zeros. Since $a > 0$, the parabola opens upward. The parabola opens upward and has two zeros, so its vertex must lie below the x-axis.

b) Identify the values of a, b, and c:

$a = 0.5$, $b = -2$, $c = 0.5$

Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$D = b^2 - 4ac$
$D = (-2)^2 - 4(0.5)(0.5)$
$D = 4 - 1$
$D = 3$

Since $D > 0$, there are two zeros. Since $a > 0$, the parabola opens upward. The parabola opens upward and has two zeros, so its vertex must lie below the x-axis.

c) Identify the values of a, b, and c:

$a = 5$, $b = -30$, $c = 45$

Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$D = b^2 - 4ac$
$D = (-30)^2 - 4(5)(45)$
$D = 900 - 900$
$D = 0$

Since $D = 0$, there is one zero. The parabola only has one zero so it must lie on the x-axis.

d) Identify the values of a, b, and c:

$a = 0.5$, $b = -4$, $c = 7.75$

Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$D = b^2 - 4ac$
$D = (-4)^2 - 4(0.5)(7.75)$
$D = 16 - 15.5$
$D = 0.5$

Since $D > 0$, there are two zeros. Since $a > 0$, the parabola opens upward. The parabola opens upward and has two zeros, so its vertex must lie below the x-axis.

10. a) First determine the discriminant. Identify the values of a, b, and c:
$a = 5$, $b = 6$, $c = k$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:
$D = b^2 - 4ac$
$D = 6^2 - 4(5)(k)$
$D = 36 - 20k$
For the equation to have two real roots, the discriminant must be positive. Since the discriminant must be positive, $36 - 20k > 0$. Solve for k in this inequality:
$36 - 20k > 0$
$36 > 20k$
$1.8 > k$
The required value of k is $k < 1.8$.
b) The discriminant is $36 - 20k$. For the equation to have one real root, the discriminant must be equal to zero. Solve for k in the equation $36 - 20k = 0$:
$36 - 20k = 0$
$36 = 20k$
$1.8 = k$
The required value of k is $k = 1.8$.
c) The discriminant is $36 - 20k$. For the equation to have no real roots, the discriminant must be negative. Since the discriminant must be negative, $36 - 20k < 0$. Solve for k in this inequality:
$36 - 20k < 0$
$36 < 20k$
$1.8 < k$
The required value of k is $k > 1.8$.
11. a) She will be standing on the platform when $t = 0$. Substitute $t = 0$ into the equation to determine her height above the water when standing on the platform:
$h = -5t^2 + t + 216$
$h = -5(0)^2 + (0) + 216$
$h = 216$
The platform is 216 m above the water.
b) If her hair touches the water, then her height will be 0 m and the corresponding quadratic equation is $0 = -5t^2 + t + 216$. Use the discriminant to show that this equation has two solutions.
Identify the values of a, b, and c:
$a = -5$, $b = 1$, $c = 216$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:
$D = b^2 - 4ac$
$D = 1^2 - 4(-5)(216)$
$D = 1 + 4320$
$D = 4321$
Since $D > 0$, the equation has two solutions.

$$t = \frac{-1 \pm \sqrt{4321}}{-10}$$

$$t \doteq \frac{-1 \pm 65.7}{-10}$$

$t \doteq 6.67$ or $t \doteq -6.67$
These solutions are $t \doteq 6.67$ and $t \doteq -6.67$. Only the positive solution makes sense in this situation. 6.67 s is the time it takes her to drop to the water.

12. First rearrange the equation so that the left side is 0:
$y = 4x^2 + 24x - 5$
$0 = 4x^2 + 24x - 5 - y$
Determine the discriminant of this equation by treating y as a constant. Identify the values of a, b, and c:
$a = 4$, $b = 24$, $c = -5 - y$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:
$D = b^2 - 4ac$
$D = 24^2 - 4(4)(-5 - y)$
$D = 576 - 16(-5 - y)$
$D = 576 + 80 + 16y$
$D = 656 + 16y$
For the equation to have no solutions, the discriminant must be negative. Solve the inequality $656 + 16y < 0$ for y to determine the required values of y:
$656 + 16y < 0$
$16y < -656$
$y < -41$
The equation has no solution when $y < -41$.
13. At the tangent points, the relations will have the same y-values. Substitute the equation of the line for the y-value of the parabola to obtain the equation $x^2 + (x + k)^2 = 25$. Simplify this equation and rearrange it so that the right side is 0:
$x^2 + (x + k)^2 = 25$
$x^2 + x^2 + 2xk + k^2 = 25$
$2x^2 + 2xk + k^2 = 25$
$2x^2 + 2xk + k^2 - 25 = 0$
Determine the discriminant by treating k as a constant. Identify the values of a, b, and c:
$a = 2$, $b = 2k$, $c = k^2 - 25$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:
$D = b^2 - 4ac$
$D = (2k)^2 - 4(2)(k^2 - 25)$
$D = 4k^2 - 8(k^2 - 25)$
$D = 4k^2 - 8k^2 + 200$
$D = -4k^2 + 200$
Since the line touches the circle at exactly one point, x can have only one real solution in the equation $2x^2 + 2xk + k^2 - 25 = 0$. For the equation to have only one solution, the discriminant, $-4k^2 + 200$, must be zero. Solve for k in the equality $-4k^2 + 200 = 0$ to determine which values of k give only one solution for x:

$$-4k^2 + 200 = 0$$
$$-4k^2 = -200$$
$$k^2 = 50$$
$$\sqrt{k^2} = \pm\sqrt{50}$$
$$k = \pm\sqrt{50}$$

The values of k are $x \doteq 7.07$ and $x \doteq -7.07$.
14. Agree. e.g., r and s are both solutions to the relation, therefore there must be two solutions and the discriminant cannot be negative.

15. Answers may vary, e.g.,
a) vertex form: $y = (x + 2)^2 - 9$;
standard form: $y = x^2 + 4x - 5$;
discriminant:
Identify the values of a, b, and c:
$a = 1$, $b = 4$, $c = -5$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$$D = b^2 - 4ac$$
$$D = 4^2 - 4(1)(-5)$$
$$D = 16 + 20$$
$$D = 36$$

Since $D > 0$, there are two zeros.
b) vertex form: $y = 2(x - 3)^2$;
standard form: $y = 2x^2 - 12x + 18$;
discriminant:
Identify the values of a, b, and c:
$a = 2$, $b = -12$, $c = 18$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$$D = b^2 - 4ac$$
$$D = (-12)^2 - 4(2)(18)$$
$$D = 144 - 144$$
$$D = 0$$

Since $D = 0$, there is one zero.
c) vertex form: $y = (x + 5)^2 + 2$;
standard form: $y = x^2 + 10x + 27$;
discriminant:
Identify the values of a, b, and c:
$a = 1$, $b = 10$, $c = 27$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$$D = b^2 - 4ac$$
$$D = 10^2 - 4(1)(27)$$
$$D = 100 - 108$$
$$D = -8$$

Since $D < 0$, there are no zeros.
16. Answers may vary, e.g.,
a) i) $y = (x - 2)(x + 6)$
ii) $y = (x + 1)(x + 2)$
iii) $y = (x - 12)(x - 3)$
b) i) $y = x^2 + 4x - 12$
ii) $y = x^2 + 3x + 2$
iii) $y = x^2 - 15x + 36$
c) i) Identify the values of a, b, and c:
$a = 1$, $b = 4$, $c = -12$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$$D = b^2 - 4ac$$
$$D = 4^2 - 4(1)(-12)$$
$$D = 16 + 48$$
$$D = 64$$

ii) Identify the values of a, b, and c:
$a = 1$, $b = 3$, $c = 2$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$$D = b^2 - 4ac$$
$$D = 3^2 - 4(1)(2)$$
$$D = 9 - 8$$
$$D = 1$$

iii) Identify the values of a, b, and c:
$a = 1$, $b = -15$, $c = 36$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$$D = b^2 - 4ac$$
$$D = (-15)^2 - 4(1)(36)$$
$$D = 225 - 144$$
$$D = 81$$

d) If the discriminant is a perfect square, then the equation is factorable.
17. At a point of intersection, the relations will have the same y-values. Substitute the equation for one parabola for the y-value of the other to obtain the equation $(x + 3)^2 = -2x^2 - 5$. Simplify this equation and rearrange it so that the right side is 0:

$$(x + 3)^2 = -2x^2 - 5$$
$$x^2 + 6x + 9 = -2x^2 - 5$$
$$3x^2 + 6x + 14 = 0$$

To determine the number of solutions to this equation, use the discriminant. Identify the values of a, b, and c:
$a = 3$, $b = 6$, $c = 14$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$$D = b^2 - 4ac$$
$$D = 6^2 - 4(3)(14)$$
$$D = 36 - 168$$
$$D = -132$$

Since $D < 0$, there are no solutions to this equation, so the relations never intersect.

Lesson 6.6: Solving Problems Using Quadratic Models, page 357

1. a) The y-coordinate of the vertex represents the maximum height of the ball and the x-coordinate represents the time when the maximum height is reached. The first zero represents the time at which the ball leaves the ground and the second zero represents the time at which the ball returns to the ground.
b) The y-coordinate of the vertex represents the maximum height of the ball and the x-coordinate represents the horizontal distance the ball is from the balcony when the maximum height is reached. The first zero has no meaning and the second zero represents the horizontal distance from the balcony where it hits the ground.
c) The y-coordinate of the vertex represents the maximum profit and the x-coordinate represents the selling price that produces this maximum profit. Both zeros represent the selling prices that ensure the company breaks even.
d) The y-coordinate of the vertex represents the minimum cost of creating items and the x-coordinate represents the number of items that should be produced to result in this minimum cost. There are no zeros because any number of items will always cost some amount to produce.
e) The y-coordinate of the vertex represents the minimum height above the ground and the x-coordinate represents the time when this minimum height is reached. There are no zeros because they would mean that the person swinging went beneath the ground.

2. a) The model rocket is on top of the school when $t = 0$. Substitute $t = 0$ into the height equation to determine the height of the school:

$$\begin{aligned} h &= 15 + 22t - 5t^2 \\ h &= 15 + 22(0) - 5(0)^2 \\ h &= 15 \end{aligned}$$

The height of the school is 15 m.

b) Substitute $h = 10$ into the height equation and rearrange it so that the left side is 0:

$$\begin{aligned} h &= 15 + 22t - 5t^2 \\ 10 &= 15 + 22t - 5t^2 \\ 0 &= 5 + 22t - 5t^2 \end{aligned}$$

This trinomial does not appear to be factorable over the set of integers. Identify the values of a, b, and c to solve the quadratic equation using the quadratic formula:

$a = -5$, $b = 22$, $c = 5$

Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-22 \pm \sqrt{22^2 - 4(-5)(5)}}{2(-5)}$$

$$t = \frac{-22 \pm \sqrt{484 + 100}}{-10}$$

$$t = \frac{-22 \pm \sqrt{584}}{-10}$$

The roots of the quadratic equation are $t \doteq 4.62$ and $t \doteq -0.22$. The time must be positive so the length of time it takes the rocket to pass the window is 4.62 s.

c) Substitute $h = 0$ into the height equation and reorder the terms:

$$\begin{aligned} h &= 15 + 22t - 5t^2 \\ 0 &= 15 + 22t - 5t^2 \\ 0 &= -5t^2 + 22t + 15 \end{aligned}$$

This is a trinomial where $a \neq 1$. Determine two numbers whose product is $(-5)(15) = -75$ and whose sum is 22. The numbers are 25 and -3. Use these to decompose the middle term and then factor:

$$\begin{aligned} -5t^2 + 22t + 15 &= -5t^2 + 25t - 3t + 15 \\ &= -5t(t - 5) - 3(t - 5) \\ &= (-5t - 3)(t - 5) \end{aligned}$$

Since $(-5t - 3)(t - 5) = 0$, either $-5t - 3 = 0$ or $t - 5 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} -5t - 3 &= 0 \qquad t - 5 = 0 \\ -5t &= 3 \qquad\quad t = 5 \\ t &= -\frac{3}{5} \end{aligned}$$

The solutions are $t = -\frac{3}{5}$ and $t = 5$. The time must be positive so the rocket hits the ground after 5 s.

d) The y-coordinate of the vertex of the relation represents the maximum height of the rocket. Determine the vertex of the relation by completing the square.

$$\begin{aligned} h &= -5t^2 + 22t + 15 \\ h &= -5(t^2 - 4.4t) + 15 \\ h &= -5(t^2 - 4.4t + 4.84 - 4.84) + 15 \\ h &= -5[(t - 2.2)^2 - 4.84] + 15 \\ h &= -5(t - 2.2)^2 + 24.2 + 15 \\ h &= -5(t - 2.2)^2 + 39.2 \end{aligned}$$

The vertex is (2.2, 39.2) so the maximum height of the rocket is 39.2 m.

3. a) The y-coordinate of the vertex of the relation represents the maximum height that the water can reach. Determine the vertex of the relation by completing the square.

$$\begin{aligned} H &= -0.011x^2 + 0.99x + 1.6 \\ H &= -0.011(x^2 - 90x) + 1.6 \\ H &= -0.011(x^2 - 90x + 2025 - 2025) + 1.6 \\ H &= -0.011[(x - 45)^2 - 2025) + 1.6 \\ H &= -0.011(x - 45)^2 + 22.275 + 1.6 \\ H &= -0.011(x - 45)^2 + 23.875 \end{aligned}$$

The vertex is (45, 23.875) so the maximum height of the water is 23.88 m.

b) Substitute $H = 15$ into the height equation and rearrange it so that the left side is 0:

$$\begin{aligned} H &= -0.011x^2 + 0.99x + 1.6 \\ 15 &= -0.011x^2 + 0.99x + 1.6 \\ 0 &= -0.011x^2 + 0.99x - 13.4 \end{aligned}$$

The zeros of this equation represent the horizontal distance from the 15 m high window the nozzle of the hose must be for the water to pass through it. Use the quadratic formula to solve for x. Identify the values of a, b, and c:

$a = -0.011$, $b = 0.99$, $c = -13.4$

Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-0.99 \pm \sqrt{0.99^2 - 4(-0.011)(-13.4)}}{2(-0.011)}$$

$$x = \frac{-0.99 \pm \sqrt{0.9801 - 0.5896}}{-0.022}$$

$$x = \frac{-0.99 \pm \sqrt{0.3905}}{-0.022}$$

The roots of the quadratic equation are $x \doteq 16.60$ and $x \doteq 73.40$, therefore the firefighter could stand either 16.60 m or 73.40 m away from the window and still have the water pass through it.

4. Model his height using the factored form of a quadratic relation, $h = a(t - r)(t - s)$, where t is the time, in seconds, h is the height above the trampoline, in metres, a is a constant, and r and s are the zeros. The zeros of this height model are $t = 0$ and $t = 2$, since each jump takes 2 s from beginning to end. Substitute these values for r and s into the factored form of the equation:

$$\begin{aligned} h &= a(t - 0)(t - 2) \\ h &= a(t^2 - 2t) \end{aligned}$$

He passes his window, which is 4 m, after 0.4 s, so substitute $h = 4$ and $t = 0.4$ into this equation to solve for a:

$$\begin{aligned} h &= a(t^2 - 2t) \\ 4 &= a[(0.4)^2 - 2(0.4)] \\ 4 &= a(0.16 - 0.8) \\ 4 &= -0.64a \\ -6.25 &= a \end{aligned}$$

The model for his height is $h = -6.25(t^2 - 2t)$. The x-coordinate of the vertex is 1, since after 1 s he is in mid-jump. Substitute $t = 1$ into the height model to determine his height at this time.

$$\begin{aligned} h &= -6.25(t^2 - 2t) \\ h &= -6.25[(1)^2 - 2(1)] \\ h &= -6.25(-1) \\ h &= 6.25 \end{aligned}$$

His maximum height is 6.25 m.

5. a) Since her maximum profit and selling price that produces it are given, use the vertex form of a quadratic relation to model her profit. The model is $P = a(x - 28)^2 + 1024$ where P is the profit, x is the selling price of each bottle, and a is a constant. She knows she will lose \$4160 if she sells the bottles for \$10, so substitute $P = -4160$ and $x = 10$ into this equation ito solve for a:

$$\begin{aligned} P &= a(x - 28)^2 + 1024 \\ -4160 &= a(10 - 28)^2 + 1024 \\ -4160 &= a(-18)^2 + 1024 \\ -4160 &= 324a + 1024 \\ -5184 &= 324a \\ -16 &= a \end{aligned}$$

The model for her profit is $P = -16(x - 28)^2 + 1024$.

b) If she breaks even, then her profit will be \$0. Substitute $P = 0$ into the profit model and solve for x to determine which selling price will ensure that she breaks even:

$$\begin{aligned} P &= -16(x - 28)^2 + 1024 \\ 0 &= -16(x - 28)^2 + 1024 \\ -1024 &= -16(x - 28)^2 \\ 64 &= (x - 28)^2 \\ \pm 8 &= x - 28 \\ 28 \pm 8 &= x \end{aligned}$$

The solutions are $x = 20$ and $x = 36$, so the required selling price is \$20 or \$36.

6. a) The halfway point is half of 21 m, which is 10.5 m. Substitute $h = 10.5$ into the height model and solve for t to determine when the performer reaches this height:

$$\begin{aligned} h &= -4.9t^2 + 21 \\ 10.5 &= -4.9t^2 + 21 \\ -10.5 &= -4.9t^2 \\ \frac{10.5}{4.9} &= t^2 \\ \pm\sqrt{\frac{10.5}{4.9}} &= t \end{aligned}$$

The solutions are $t \doteq -1.46$ and $t \doteq 1.46$. The time must be positive so it takes the performer about 1.46 s to reach the halfway point.

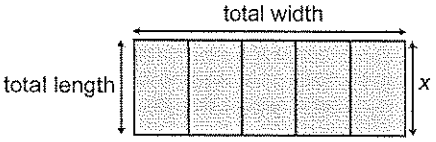
b) When the performer hits the water, height is 0 m. Substitute $h = 0$ into the equation and solve for t to determine when the performer reaches the water:

$$\begin{aligned} h &= -4.9t^2 + 21 \\ 0 &= -4.9t^2 + 21 \\ -21 &= -4.9t^2 \\ \frac{21}{4.9} &= t^2 \\ \pm\sqrt{\frac{21}{4.9}} &= t \end{aligned}$$

The solutions are $t \doteq -2.07$ and $t \doteq 2.07$. The time must be positive so it takes the performer about 2.07 s to reach the water.

c) Answers may vary, e.g., gravity causes the diver to accelerate toward the water, resulting in a non-linear relation.

7. Let x represent the length of each pen, which is also the total length of the enclosure.



The total length of fence is 30 m, the total fence used for the lengths of all the pens is $6x$, and the total width of the enclosure remains, of which there are two sides. From this information, the total width of the enclosure is $\frac{30 - 6x}{2}$ or $15 - 3x$. The area of the enclosure is the total width multiplied by the total length or $A = x(15 - 3x)$. Use partial factoring to determine that the area is maximized when $x = 2.5$. Substitute $x = 2.5$ into the expression for the total width of the enclosure:

$$\begin{aligned} \frac{30 - 6(2.5)}{2} &= \frac{30 - 15}{2} \\ &= 7.5 \end{aligned}$$

The total width of the enclosure is 7.5 m and the total length of the enclosure is 2.5 m so the total area is $2.5 \text{ m} \times 7.5 \text{ m} = 18.75 \text{ m}^2$ when the area is maximized.

8. a) In 1999, the number of years since 1999 is 0. Substitute $x = 0$ in the population model to determine the number of deer in the park at this time:

$$\begin{aligned} P &= 8x^2 - 112x + 570 \\ P &= 8(0)^2 - 112(0) + 570 \\ P &= 570 \end{aligned}$$

By the mode,l there were 570 deer in the park in 1999.

b) The coordinates of the vertex represent the year the deer population was at a minimum and the number of deer in the park at this time. Determine the vertex by completing the square:

$$\begin{aligned} P &= 8x^2 - 112x + 570 \\ P &= 8(x^2 - 14x) + 570 \\ P &= 8(x^2 - 14x + 49 - 49) + 570 \\ P &= 8[(x - 7)^2 - 49] + 570 \\ P &= 8(x - 7)^2 - 392 + 570 \\ P &= 8(x - 7)^2 + 178 \end{aligned}$$

The vertex is (7, 178) so the minimum population of deer occurred 7 years after 1999, in 2006, and the number of deer in the park at this time was 178.

c) Substitute $P = 0$ into the population model to see if there is ever a year in which this occurs:
$P = 8x^2 - 112x + 570$
$0 = 8x^2 - 112x + 570$
Use the discriminant to determine how many solutions this quadratic equation has. Identify the values of a, b, and c: $a = 8$, $b = -112$, $c = 570$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:

$$\begin{aligned} D &= b^2 - 4ac \\ D &= (-112)^2 - 4(8)(570) \\ D &= 12\,544 - 18\,240 \\ D &= -5696 \end{aligned}$$

$D < 0$, so the quadratic equation has no solutions. This indicates that the deer population will never reach zero according to the model.
d) Answers may vary, e.g., probably not, since the curve continues to increase after 2006. According to the model, in 2020 there would be 1746 deer, so this prediction is only useful if the deer population is expected to continue at this high rate.
9. a) When she both enters the water and returns to the surface her depth will be 0. Substitute $d = 0$ into the depth equation to determine the two times when she is at the surface of the water:

$$\begin{aligned} d &= 0.1t^2 - 3.5t + 6 \\ 0 &= 0.1t^2 - 3.5t + 6 \end{aligned}$$

Use the quadratic formula to solve for t.
Identify the values of a, b, and c:
$a = 0.1$, $b = -3.5$, $c = 6$
Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(-3.5) \pm \sqrt{(-3.5)^2 - 4(0.1)(6)}}{2(0.1)}$$

$$t = \frac{3.5 \pm \sqrt{12.25 - 2.4}}{0.2}$$

$$t = \frac{3.5 \pm \sqrt{9.85}}{0.2}$$

The solutions are $t \doteq 1.81$ and $t \doteq 33.19$. She entered the water after 1.81 s and returned to the surface after 33.19 s, so her total time spent underwater is 33.19 s − 1.81 s = 31.38 s.
b) The y-coordinate of the vertex represents the maximum depth that she dives to. Determine the vertex by completing the square:

$$\begin{aligned} d &= 0.1t^2 - 3.5t + 6 \\ d &= 0.1(t^2 - 35t) + 6 \\ d &= 0.1(t^2 - 35t + 306.25 - 306.25) + 6 \\ d &= 0.1[(t - 17.5)^2 - 306.25] + 6 \\ d &= 0.1(t - 17.5)^2 - 30.625 + 6 \\ d &= 0.1(t - 17.5)^2 - 24.625 \end{aligned}$$

The vertex is (17.5, −24.625), so she reaches a maximum depth of about 24.63 m.

10. a) The coordinates of the vertex represent the minimum cost and the number of items that should be produced that results in this minimum cost. Determine the vertex by completing the square:

$$\begin{aligned} C &= 0.01x^2 - 1.5x + 93.25 \\ C &= 0.01(x^2 - 150x) + 93.25 \\ C &= 0.01(x^2 - 150x + 5625 - 5625) + 93.25 \\ C &= 0.01[(x - 75)^2 - 5625] + 93.25 \\ C &= 0.01(x - 75)^2 - 56.25 + 93.25 \\ C &= 0.01(x - 75)^2 + 37 \end{aligned}$$

The vertex is (75, 37), so the minimum cost of $37/h will be the result of producing 75 items each hour.
b) Substitute $C = 53$ into the cost equation to determine the production rate that results in a cost of exactly $53/h:

$$\begin{aligned} C &= 0.01x^2 - 1.5x + 93.25 \\ 53 &= 0.01x^2 - 1.5x + 93.25 \end{aligned}$$

Rearrange the equation so that the right side is 0:

$$0 = 0.01x^2 - 1.5x + 40.25$$

Solve for x using the quadratic formula. Identify the values of a, b, and c: $a = 0.01$, $b = -1.5$, $c = 40.25$
Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1.5) \pm \sqrt{(-1.5)^2 - 4(0.01)(40.25)}}{2(0.01)}$$

$$x = \frac{1.5 \pm \sqrt{2.25 - 1.61}}{0.02}$$

$$x = \frac{1.5 \pm \sqrt{0.64}}{0.02}$$

The solutions are $x \doteq 35$ and $x \doteq 115$. The cost will be exactly $53 when 35 or 115 items are produced per hour and the minimum cost occurs between these two values. This indicates that a production rate between 35 and 115 items per hour will keep the cost below $53.
11. Let x represent the width of the border. The area of each strip of border touching the garden is $3x$ and there are 3 such strips, so the total area of border touching the garden is $3 \times 3x = 9x$. The corner pieces each have an area of x^2 and there are two such pieces, so the total area of the corners is $2 \times x^2 = 2x^2$. The total area of the border is $2x^2 + 9x$ and the total area of the garden is 9. Since these areas are to be equal, set the two expressions equal to each other and solve for x to determine the width of the border.

$$\begin{aligned} 2x^2 + 9x &= 9 \\ 2x^2 + 9x - 9 &= 0 \end{aligned}$$

Solve for x using the quadratic formula. Identify the values of a, b, and c:
$a = 2$, $b = 9$, $c = -9$

Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-9 \pm \sqrt{9^2 - 4(2)(-9)}}{2(2)}$$

$$x = \frac{-9 \pm \sqrt{81+72}}{4}$$

$$x = \frac{-9 \pm \sqrt{153}}{4}$$

The solutions are $x \doteq -5.34$ and $x \doteq 0.84$. The width must be positive so the width that makes the areas equal is about 0.84 m.

12. First, determine a quadratic relation that models the ball's height. Since the ball's maximum height and the horizontal distance where it occurs are given, use the vertex form of a quadratic relation to model the ball's height. The model is $h = a(x-3)^2 + 4.2$ where h is the height, x is the horizontal distance from the jungle gym, and a is a constant. The ball reached a height of 1.5 m when it was 0 m from the jungle gym, so substitute $h = 1.5$ and $x = 0$ into this equation to solve for a:

$$\begin{aligned} h &= a(x-3)^2 + 4.2 \\ 1.5 &= a(0-3)^2 + 4.2 \\ 1.5 &= 9a + 4.2 \\ -2.7 &= 9a \\ -0.3 &= a \end{aligned}$$

The model for the ball's height is $h = -0.3(x-3)^2 + 4.2$. To determine where the ball hit the ground, determine the zeros of this relation. First expand and simplify the corresponding quadratic equation:

$$\begin{aligned} 0 &= -0.3(x-3)^2 + 4.2 \\ 0 &= -0.3(x^2 - 6x + 9) + 4.2 \\ 0 &= -0.3x^2 + 1.8x - 2.7 + 4.2 \\ 0 &= -0.3x^2 + 1.8x + 1.5 \end{aligned}$$

Solve for x using the quadratic formula. Identify the values of a, b, and c: $a = -0.3$, $b = 1.8$, $c = 1.5$

Substitute the values of a, b, and c into the quadratic formula and evaluate:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1.8 \pm \sqrt{1.8^2 - 4(-0.3)(1.5)}}{2(-0.3)}$$

$$x = \frac{-1.8 \pm \sqrt{3.24+1.8}}{-0.6}$$

$$x = \frac{-1.8 \pm \sqrt{5.04}}{-0.6}$$

The zeros are $x \doteq -0.74$ and $x \doteq 6.74$. The distance from the jungle gym must be positive, so the ball hit the ground at a distance of about 6.74 m from the jungle gym.

13. Let x represent the smallest even integer. Now, the other two integers are represented by $x + 2$ and $x + 4$. The sum of these integers squared is 980:

$$x^2 + (x+2)^2 + (x+4)^2 = 980$$

Expand and simplify this equation, and rearrange it so that the right side is 0:

$$\begin{aligned} x^2 + x^2 + 4x + 4 + x^2 + 8x + 16 &= 980 \\ 3x^2 + 12x + 20 &= 980 \\ 3x^2 + 12x - 960 &= 0 \end{aligned}$$

Factor out the common factor 3 and then factor the resulting trinomial:

$$\begin{aligned} 3(x^2 + 4x - 320) &= 0 \\ 3(x-16)(x+20) &= 0 \end{aligned}$$

Since $3(x-16)(x+20) = 0$, either $x - 16 = 0$ or $x + 20 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} x - 16 &= 0 & x + 20 &= 0 \\ x &= 16 & x &= -20 \end{aligned}$$

The solutions are $x = 16$ and $x = -20$, therefore the two sets of three integers whose squares sum to 980 are 16, 18, 20 and $-20, -18, -16$.

14. a) The ball is at a height of 0 m when it is 0 m from her. It is kicked upward and falls back down to a height of 0 m at a distance 34 m from her. Since both zeros are known, model the ball's height using the factored form of a quadratic relation, $y = a(x-r)(x-s)$, where x is the horizontal distance from her, y is the height above the ground, a is a constant, and r and s are the zeros. The zeros of this height model are $x = 0$ and $x = 34$. Substitute these values for r and s into the factored form of the equation:

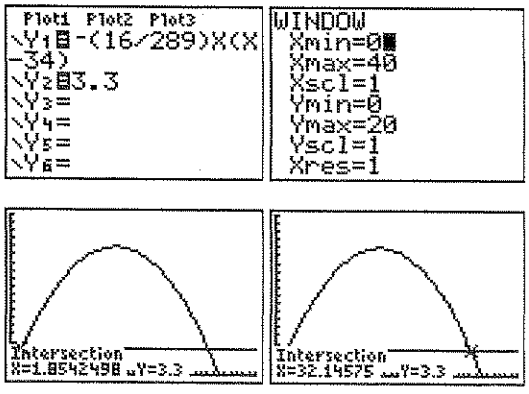
$$\begin{aligned} y &= a(x-0)(x-34) \\ y &= ax(x-34) \end{aligned}$$

The ball reaches its maximum height, 16 m, halfway between the zeros, at 17 m. Substitute $y = 16$ and $x = 17$ into this equation to solve for a:

$$\begin{aligned} y &= ax(x-34) \\ 16 &= a(17)(17-34) \\ 16 &= 17a(-17) \\ 16 &= -289a \\ -\frac{16}{289} &= a \end{aligned}$$

The equation that models the ball's height is $y = -\frac{16}{289}x(x-34)$.

b) Using a graphing calculator, graph both the height equation $y = -\frac{16}{289}x(x-34)$ and $y = 3.3$ to determine where the ball reaches a height of 3.3 m. Use the intersect operation to locate the intersection points of the two graphs.



The points of intersection are $x \doteq 1.85$ and $x \doteq 32.15$, so the ball is higher than 3.3 m when it is between about 1.85 m and 32.15 m away from her. She can stand between about 1.85 m and 32.15 m away from the uprights and still have a chance to score a field goal.

15. Answers may vary, e.g.,

a) The sum of two integers is 11. The sum of their squares is 61. Determine the integers.

b) Sean is practising skateboarding in a parabolic half-pipe. At one point, he has travelled 1.5 horizontally and 2.5 m below the lip. If the half pipe is 15.0 m across, what is the vertical distance from the lip to the bottom?

16. Let x represent the side length of the triangle. The total length of mesh is 30 m and there are three sides of length x. From this information, the length of each remaining side is $\frac{30-3x}{2}$. The area of the rectangle is then $\frac{x(30-3x)}{2}$. Drop a perpendicular bisector from the top vertex of the triangle to divide the equilateral triangle into two right triangles. Use the Pythagorean theorem to solve for the height.

$$h^2 + \left(\frac{x}{2}\right)^2 = x^2$$

$$h^2 = x^2 - \left(\frac{x}{2}\right)^2$$

$$h = \sqrt{x^2 - \left(\frac{x}{2}\right)^2}$$

$$h = \sqrt{x^2 - \frac{x^2}{4}}$$

$$h = \frac{\sqrt{3}}{2}x$$

The base is x units long, thus the area of the triangle is $\frac{\sqrt{3}}{4}x^2$. The total area is the area of the triangle added to the area of the rectangle, or $A = \frac{\sqrt{3}}{4}x^2 + \frac{x(30-3x)}{2}$.

Expanding the brackets, we get $A = \frac{\sqrt{3}}{4}x^2 + 15x - \frac{3}{2}x^2$.

Factoring again, we get $A = x\left[\left(\frac{\sqrt{3}}{4} - \frac{3}{2}\right)x + 15\right]$.

Thus, the zeros occur at $x = 0$ and $x = -\frac{15}{\frac{\sqrt{3}}{4} - \frac{3}{2}}$.

The maximum area occurs at the vertex.

The x-coordinate at the vertex is midway between the zero, so $x = \left(-\frac{15}{\frac{\sqrt{3}}{4} - \frac{3}{2}} + 0\right) \div 2 \doteq 7.0$. Solving for the remaining side, $\frac{30-3(7)}{2} = \frac{30-21}{2} = 4.5$. Thus the dimensions of the rectangle will be about 7.0 m by 4.5 m, and the triangle will be about 7.0 m for each side.

17. Let x represent the length of the field. Since the total distance he walked was 36 m, and after walking diagonally across the field and along its length he walked a total of $15 + x$ m, the width of the field is $36 - (15 + x)$ or $21 - x$. The length, width, and diagonal of the field are related by the Pythagorean theorem, $a^2 + b^2 = c^2$. Substitute $a = x$, $b = 21 - x$, and $c = 15$ into the equation and solve for x to determine the length of the field:

$$\begin{aligned} a^2 + b^2 &= c^2 \\ x^2 + (21 - x)^2 &= 15^2 \\ x^2 + 441 - 42x + x^2 &= 225 \\ 2x^2 - 42x + 441 &= 225 \\ 2x^2 - 42x + 216 &= 0 \\ 2(x^2 - 21x + 108) &= 0 \\ 2(x - 9)(x - 12) &= 0 \end{aligned}$$

Since $2(x - 9)(x - 12) = 0$, either $x - 9 = 0$ or $x - 12 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} x - 9 &= 0 & x - 12 &= 0 \\ x &= 9 & x &= 12 \end{aligned}$$

The solutions are $x = 9$ and $x = 12$. Substitute each value into the expression for the width, $21 - x$, to determine the dimensions of the field:

$$\begin{aligned} 21 - (9) &= 12 \\ 21 - (12) &= 9 \end{aligned}$$

In both cases, the dimensions of the field are 9 m by 12 m, so the only solution is 9 m by 12 m.

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1. a) Since $(2x - 5)(3x + 8) = 0$, either $2x - 5 = 0$ or $3x + 8 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} 2x - 5 &= 0 & 3x + 8 &= 0 \\ 2x &= 5 & 3x &= -8 \\ x &= \frac{5}{2} & x &= -\frac{8}{3} \end{aligned}$$

The solutions are $x = \frac{5}{2}$ and $x = -\frac{8}{3}$.

b) To determine this trinomial's factors, determine two numbers whose product is 32 and whose sum is 12. The numbers are 8 and 4, so the factored equation is $(x + 8)(x + 4) = 0$. It must be true that either $x + 8 = 0$ or $x + 4 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} x + 8 &= 0 & x + 4 &= 0 \\ x &= -8 & x &= -4 \end{aligned}$$

The solutions are $x = -8$ and $x = -4$.

c) Determine two numbers whose product is (3)(–8) = –24 and whose sum is –10. The numbers are –12 and 2. Use these to decompose the middle term and then factor:

$$\begin{aligned} 3x^2 - 10x - 8 &= 3x^2 - 12x + 2x - 8 \\ &= 3x(x-4) + 2(x-4) \\ &= (3x+2)(x-4) \end{aligned}$$

Since $(3x + 2)(x – 4) = 0$, either $3x + 2 = 0$ or $x – 4 = 0$. Solve the two equations to obtain both solutions:

$$\begin{aligned} 3x + 2 &= 0 & x - 4 &= 0 \\ 3x &= -2 & x &= 4 \\ x &= -\frac{2}{3} \end{aligned}$$

The solutions are $x = -\frac{2}{3}$ and $x = 4$.

d) Rearrange the equation so that the right side is 0. Combine like terms.

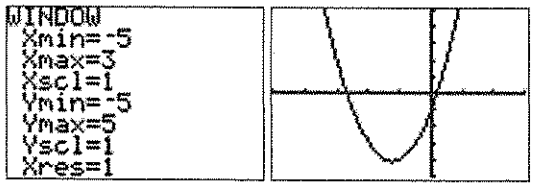
$$\begin{aligned} 3x^2 - 5x + 5 &= 2x^2 + 4x - 3 \\ 3x^2 - 5x + 5 - 2x^2 - 4x + 3 &= 0 \\ x^2 - 9x + 8 &= 0 \end{aligned}$$

To determine this trinomial's factors, determine two numbers whose product is 8 and whose sum is –9. The numbers are –8 and –1, so the factored equation is $(x – 8)(x – 1) = 0$. It must be true that either $x – 8 = 0$ or $x – 1 = 0$. Solve the two equations to obtain both solutions:

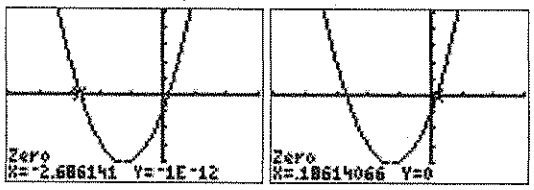
$$\begin{aligned} x - 1 &= 0 & x - 8 &= 0 \\ x &= 1 & x &= 8 \end{aligned}$$

The solutions are $x = 1$ and $x = 8$.

e) I graphed $y = 2x^2 + 5x – 1$ using these window settings.



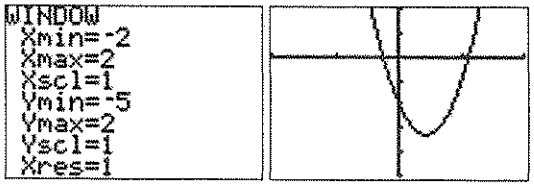
Using the Zero operation of the calculator, I estimated that the x-intercepts are $x \doteq –2.69$ and $x \doteq 0.19$.



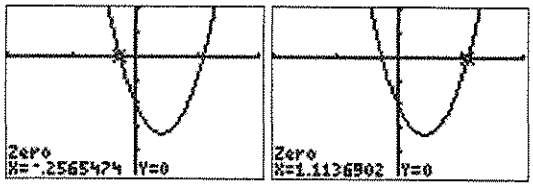
f) Rearrange the equation so that the right side is 0. Expand and combine like terms.

$$\begin{aligned} 5x(x-1) + 5 &= 7 + x(1 - 2x) \\ 5x(x-1) + 5 - 7 - x(1 - 2x) &= 0 \\ 5x^2 - 5x + 5 - 7 - x + 2x^2 &= 0 \\ 7x^2 - 6x - 2 &= 0 \end{aligned}$$

I graphed $y = 7x^2 – 6x – 2$ using these window settings.



Using the Zero operation of the calculator, I estimated that the x-intercepts are $x \doteq –0.26$ and $x \doteq 1.11$.



The solutions are about –0.26 and 1.11.

2. a) Substitute $v = 12$ into the equation and solve.

$$\begin{aligned} d &= 0.002(2v^2 + 10v + 3000) \\ d &= 0.002(2(12)^2 + 10(12) + 3000) \\ d &= 0.002(288 + 120 + 3000) \\ d &= 6.816 \text{ m} \end{aligned}$$

The safe stopping distance for a boat travelling at 12 km/h is about 6.82 m

b) Let $d = 15$ and then rearrange the equation so that the left side is 0.

$$\begin{aligned} d &= 0.002(2v^2 + 10v + 3000) \\ 15 &= 0.002(2v^2 + 10v + 3000) \\ 0 &= 0.002(2v^2 + 10v + 3000) - 15 \end{aligned}$$

Solve this equation to determine v, the initial speed of the boat. Factor the trinomial.

$$\begin{aligned} 0 &= 0.002(2v^2 + 10v + 3000) - 15 \\ 0 &= 0.002(2v^2 + 10v + 3000 - 7500) \\ 0 &= 0.002(2v^2 + 10v - 4500) \\ 0 &= 0.004(v^2 + 5v - 2250) \\ 0 &= 0.004(v + 50)(v - 45) \end{aligned}$$

Since $(v + 50)(v – 45) = 0$, either $v + 50 = 0$ or $v – 45 = 0$.

Solve the two equations to obtain both solutions:

$$\begin{aligned} v + 50 &= 0 & v - 45 &= 0 \\ v &= -50 & v &= 45 \end{aligned}$$

The boat cannot have a negative speed. Therefore, the initial speed of the boat was 45 km/h.

3. Each trinomial is in the form $x^2 + bx + c$. To create a perfect-square trinomial, use the coefficient of x, b, to solve $c = \left(\frac{b}{2}\right)^2$.

a) The coefficient of x, b, is 8. Determine c:

$$\begin{aligned} c &= \left(\frac{b}{2}\right)^2 \\ c &= 4^2 \\ c &= 16 \end{aligned}$$

b) The coefficient of x, b, is –16. Determine c:

$$\begin{aligned} c &= \left(\frac{b}{2}\right)^2 \\ c &= (-8)^2 \\ c &= 64 \end{aligned}$$

c) The coefficient of x, b, is 19. Determine c:

$$\begin{aligned} c &= \left(\frac{b}{2}\right)^2 \\ c &= 9.5^2 \\ c &= 90.25 \end{aligned}$$

d) The coefficient of x^2, a, is 2. The coefficient of x is 12, and b equals this coefficient divided by a:

$b = 12 \div 2$

$b = 6$

Determine c:

$$c = \left(\frac{b}{2}\right)^2$$

$c = 3^2$

$c = 9$

c must be multiplied by a, so $c = 18$.

e) The coefficient of x^2, a, is -3. The coefficient of x is 15, and b equals this coefficient divided by a:

$b = 15 \div -3$

$b = -5$

Determine c:

$$c = \left(\frac{b}{2}\right)^2$$

$c = (-2.5)^2$

$c = 6.25$

c must be multiplied by a, so $c = -18.75$.

f) The coefficient of x^2, a, is 0.1. The coefficient of x is -7, and b equals this coefficient divided by a:

$b = -7 \div 0.1$

$b = -70$

Determine c:

$$c = \left(\frac{b}{2}\right)^2$$

$c = (-35)^2$

$c = 1225$

c must be multiplied by a, so $c = 122.5$.

4. a) The square of half of the coefficient of the x term is $\left(\frac{8}{2}\right)^2 = 16$. Add and subtract this to the relation and then factor to complete the square:

$y = x^2 + 8x - 2$

$y = x^2 + 8x - 2 + 16 - 16$

$y = (x + 4)^2 - 18$

b) The square of half of the coefficient of the x term is $\left(\frac{-20}{2}\right)^2 = 100$. Add and subtract this to the relation and then factor to complete the square:

$y = x^2 - 20x + 95$

$y = x^2 - 20x + 95 + 100 - 100$

$y = (x - 10)^2 - 5$

c) First, factor out -3 from the first two terms:

$y = -3x^2 + 12x - 2$

$y = -3(x^2 - 4x) - 2$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is $\left(\frac{-4}{2}\right)^2 = 4$:

$y = -3(x^2 - 4x + 4 - 4) - 2$

Finish completing the square by factoring the trinomial and then simplifying:

$y = -3[(x - 2)^2 - 4] - 2$

$y = -3(x - 2)^2 + 12 - 2$

$y = -3(x - 2)^2 + 10$

d) First, factor out 0.2 from the first two terms:

$y = 0.2x^2 - 0.4x + 1$

$y = 0.2(x^2 - 2x) + 1$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is $\left(\frac{-2}{2}\right)^2 = 1$:

$y = 0.2(x^2 - 2x + 1 - 1) + 1$

Finish completing the square by factoring the trinomial and then simplifying:

$y = 0.2[(x - 1)^2 - 1] + 1$

$y = 0.2(x - 1)^2 - 0.2 + 1$

$y = 0.2(x - 1)^2 + 0.8$

e) First, factor out 2 from the first two terms:

$y = 2x^2 + 10x - 12$

$y = 2(x^2 + 5x) - 12$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is $\left(\frac{5}{2}\right)^2 = 6.25$:

$y = 2(x^2 + 5x + 6.25 - 6.25) - 12$

Finish completing the square by factoring the trinomial and then simplifying:

$y = 2[(x + 2.5)^2 - 6.25] - 12$

$y = 2(x + 2.5)^2 - 12.5 - 12$

$y = 2(x + 2.5)^2 - 24.5$

f) First, factor out -4.9 from the first two terms:

$y = -4.9x^2 - 19.6x + 12$

$y = -4.9(x^2 + 4x) + 12$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is $\left(\frac{4}{2}\right)^2 = 4$:

$y = -4.9(x^2 + 4x + 4 - 4) + 12$

Finish completing the square by factoring the trinomial and then simplifying:

$y = -4.9[(x + 2)^2 - 4] + 12$

$y = -4.9(x + 2)^2 + 19.6 + 12$

$y = -4.9(x + 2)^2 + 31.6$

5. a) First, factor out -3 from the first two terms:

$y = -3x^2 - 12x - 2$

$y = -3(x^2 + 4x) - 2$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is $\left(\frac{4}{2}\right)^2 = 4$:

$y = -3(x^2 + 4x + 4 - 4) - 2$

Finish completing the square by factoring the trinomial and then simplifying:

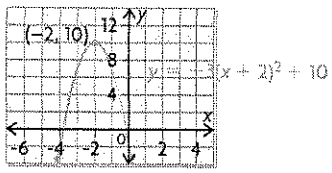
$y = -3[(x + 2)^2 - 4] - 2$

$y = -3(x + 2)^2 + 12 - 2$

$y = -3(x + 2)^2 + 10$

b) Stretch $y = x^2$ by a factor of 3, translate the parabola 2 units left and 10 units up, and then reflect the parabola in the x-axis.

c)



6. To determine the maximum height of the ball, write the equation in vertex form.

First, factor out –0.2 from the first two terms:

$y = -0.2x^2 + 2.4x + 2$

$y = -0.2(x^2 - 12x) + 2$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is

$\left(\frac{-12}{2}\right)^2 = 36$:

$y = -0.2(x^2 - 12x + 36 - 36) + 2$

Finish completing the square by factoring the trinomial and then simplifying:

$y = -0.2[(x - 6)^2 - 36] + 2$

$y = -0.2(x - 6)^2 + 7.2 + 2$

$y = -0.2(x - 6)^2 + 9.2$

Since $a > 0$, the y-coordinate of the vertex is the maximum value of the relation. The maximum height of the ball is 9.2 m.

7. The space will be rectangular and fenced on three sides. The perimeter will be:

$l + l + w = 46$

$2l + w = 46$

$w = 46 - 2l$

The area will be:

$A = lw$

$A = l(46 - 2l)$

$A = -2l^2 + 46l$

Complete the square to determine the length, l, that results in the maximum area. This is represented by the l-coordinate of the vertex:

$A = -2l^2 + 46l$

$A = -2(l^2 - 23l)$

$A = -2(l^2 - 23l + 132.25 - 132.25)$

$A = -2[(l - 11.5)^2 - 132.25]$

$A = -2(l - 11.5)^2 + 264.5$

To produce the maximum area, the length of the meditation space should be 11.5 m. Calculate the width.

$w = 46 - 2l$

$w = 46 - 2(11.5)$

$w = 23.0$

Cam should make the rectangular meditation space 11.5 m by 23.0 m to have the maximum area.

8. a) First, factor out 3 from the first two terms:

$0 = 3x^2 - 4x - 10$

$0 = 3\left(x^2 - \frac{4}{3}x\right) - 10$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is

$\left(-\frac{4}{3} \div 2\right)^2 = \frac{4}{9}$:

$0 = 3\left(x^2 - \frac{4}{3}x + \frac{4}{9} - \frac{4}{9}\right) - 10$

$0 = 3\left[\left(x - \frac{2}{3}\right)^2 - \frac{4}{9}\right] - 10$

$0 = 3\left(x - \frac{2}{3}\right)^2 - \frac{4}{3} - 10$

$0 = 3\left(x - \frac{2}{3}\right)^2 - 11\frac{1}{3}$

Solve for x.

$11\frac{1}{3} = 3\left(x - \frac{2}{3}\right)^2$

$3\frac{7}{9} = \left(x - \frac{2}{3}\right)^2$

$\pm\sqrt{3\frac{7}{9}} = x - \frac{2}{3}$

$\frac{2}{3} \pm \sqrt{3\frac{7}{9}} = x$

The solutions are about $x = 2.61$ and $x = -1.28$.

b) Combine like terms so the right side equals 0. Factor out –4. The b value in this trinomial equals 0.

$-4x^2 + 1 = -15$

$-4x^2 + 16 = 0$

$-4(x^2 - 4) = 0$

$-4(x - 2)(x + 2) = 0$

Solve for both solutions for x.

$x + 2 = 0$ $\qquad$ $x - 2 = 0$

$x = -2$ $\qquad$ $x = 2$

The solutions are $x = -2$ and $x = 2$.

c) Rearrange the equation so the right side equals 0.

$x^2 = 6x + 10$

$x^2 - 6x - 10 = 0$

This equation is difficult to factor. Use the quadratic formula to determine the values of x, where $a = 1$, $b = -6$, and $c = -10$.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-10)}}{2(1)}$

$x = \frac{6 \pm \sqrt{36 + 40}}{2}$

$x = \frac{6 \pm \sqrt{76}}{2}$

The solutions are about $x = -1.36$ and $x = 7.36$.

d) The equation is already in vertex form. Solve for x.

$$(x-3)^2-4=0$$
$$(x-3)^2=4$$
$$x-3=\pm\sqrt{4}$$
$$x=3\pm\sqrt{4}$$

The solutions are $x = 1$ and $x = 5$.

e) Expand and combine like terms on the left side of the equation.

$$(2x+5)(3x-2) = (x+1)$$
$$6x^2-4x+15x-10-x-1 = 0$$
$$6x^2+10x-11 = 0$$

Use the quadratic formula to determine the values of x, where $a = 6$, $b = 10$, and $c = -11$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-10\pm\sqrt{10^2-4(6)(-11)}}{2(6)}$$
$$x=\frac{-10\pm\sqrt{100+264}}{12}$$
$$x=\frac{-10\pm\sqrt{364}}{12}$$

The solutions are about $x = -2.42$ and $x = 0.76$.

f) First, factor out 1.5 from the first two terms:

$$1.5x^2-6.1x+1.1=0$$
$$1.5(x^2-4.07x)+1.1=0$$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is

$$\left(\frac{-4.07}{2}\right)^2=4.14:$$
$$0 = 1.5(x^2-4.07x+4.14-4.14)+1.1$$
$$0 = 1.5[(x-2.04)^2-4.14]+1.1$$
$$0 = 1.5(x-2.04)^2-6.2+1.1$$
$$0 = 1.5(x-2.04)^2-5.1$$

Solve for x.

$$1.5(x-2.04)^2=5.1$$
$$(x-2.04)^2=3.4$$
$$x-2.04=\pm\sqrt{3.4}$$
$$x=2.04\pm\sqrt{3.4}$$

The solutions are about $x = 0.19$ and $x = 3.88$.

9. Write the equation in vertex form. First, factor out –0.1 from the first two terms

$$h=-0.1x^2+2.4x+8.1$$
$$h=-0.1(x^2-24x)+8.1$$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is

$$\left(\frac{-24}{2}\right)^2=144:$$
$$h=-0.1(x^2-24x+144-144)+8.1$$
$$h=-0.1[(x-12)^2-144]+8.1$$
$$h=-0.1(x-12)^2+14.4+8.1$$
$$h=-0.1(x-12)^2+22.5$$

Let $h = 10$ and solve for x to determine the distance the balloon has travelled.

$$10=-0.1(x-12)^2+22.5$$
$$10-22.5=-0.1(x-12)^2$$
$$-12.5=-0.1(x-12)^2$$
$$125=(x-12)^2$$
$$\pm\sqrt{125}=x-12$$
$$12\pm\sqrt{125}=x$$

The solutions are about $x = 0.82$ and $x = 23.18$.

The water balloon has travelled about 0.82 m or 23.18 m.

10. Write the equation in vertex form. First, factor out 0.0025 from the first two terms

$$h=0.0025x^2-0.9x+120$$
$$h=0.0025(x^2-360x)+120$$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is

$$\left(\frac{-360}{2}\right)^2=32\,400:$$
$$h=0.0025(x^2-360x+32\,400-32\,400)+120$$
$$h=0.0025[(x-180)^2-32\,400]+120$$
$$h=0.0025(x-180)^2-81+120$$
$$h=0.0025(x-180)^2+39$$

Let $h = 50$ and solve for x to determine the distance the chair is from the post.

$$50=0.0025(x-180)^2+39$$
$$50-39=0.0025(x-180)^2$$
$$11=0.0025(x-180)^2$$
$$4400=(x-180)^2$$
$$\pm\sqrt{4400}=x-180$$
$$180\pm\sqrt{4400}=x$$

The solutions are about $x = 113.67$ and $x = 246.33$.

The chair is about 133.67 cm or 246.33 cm from the pole.

11. a) Identify the values of a, b, and c:
$a = 2, b = -5, c = 1$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:
$D = b^2 - 4ac$
$D = (-5)^2 - 4(2)(1)$
$D = 25 - 8$
$D = 17$
Since $D > 0$, there are two real solutions.
b) Identify the values of a, b, and c:
$a = -3.5, b = -2.1, c = -1$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:
$D = b^2 - 4ac$
$D = (-2.1)^2 - 4(-3.5)(-1)$
$D = 4.41 - 14$
$D = -9.59$
Since $D < 0$, there are no real solutions.
c) Identify the values of a, b, and c:
$a = 1, b = 5, c = 8$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:
$D = b^2 - 4ac$
$D = (5)^2 - 4(1)(8)$
$D = 25 - 32$
$D = -7$
Since $D < 0$, there are no real solutions.
d) Identify the values of a, b, and c:
$a = 4, b = 0, c = -15$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:
$D = b^2 - 4ac$
$D = (0)^2 - 4(4)(-15)$
$D = 240$
Since $D > 0$, there are two real solutions.
e) Expand and combine like terms on the left side of the equation.
$5(x^2 + 2x + 5) = -2(2x - 25)$
$5x^2 + 10x + 25 + 4x - 50 = 0$
$5x^2 + 14x - 25 = 0$
Identify the values of a, b, and c:
$a = 5, b = 14, c = -25$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:
$D = b^2 - 4ac$
$D = (14)^2 - 4(5)(-25)$
$D = 196 + 500$
$D = 696$
Since $D > 0$, there are two real solutions.
12. a) Expand and combine like terms.
$y = (x - 4)(2x + 9)$
$y = 2x^2 + 9x - 8x - 36$
$y = 2x^2 + x - 36$
Identify the values of a, b, and c:
$a = 2, b = 1, c = -36$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:
$D = b^2 - 4ac$
$D = (1)^2 - 4(2)(-36)$
$D = 1 + 288$
$D = 289$
Since $D > 0$, the relation has two x-intercepts.
b) The relation is written in vertex form. The vertex of the relation is (3, 2), and the parabola opens downward since $a < 0$. Since the vertex is above the x-axis and the parabola opens downward, the relation has two x-intercepts.
c) Identify the values of a, b, and c:
$a = 2, b = 8, c = 14$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:
$D = b^2 - 4ac$
$D = 8^2 - 4(2)(14)$
$D = 64 - 112$
$D = -48$
Since $D < 0$, the relation has no x-intercepts.
d) Expand.
$y = 2x(x - 5) + 7$
$y = 2x^2 - 10x + 7$
Identify the values of a, b, and c:
$a = 2, b = -10, c = 7$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:
$D = b^2 - 4ac$
$D = (-10)^2 - 4(2)(7)$
$D = 100 - 56$
$D = 44$
Since $D > 0$, the relation has two x-intercepts.
e) Identify the values of a, b, and c:
$a = -1.4, b = -4, c = -5.4$
Substitute the values of a, b, and c into the expression for the discriminant and evaluate:
$D = b^2 - 4ac$
$D = (-4)^2 - 4(-1.4)(-5.4)$
$D = 16 - 30.24$
$D = -14.24$
Since $D < 0$, the relation has no x-intercepts.
13. a) Substitute $t = 10$ into the equation and solve for H.
$H = 3500 - 5t^2$
$H = 3500 - 5(10)^2$
$H = 3500 - 500$
$H = 3000$ m
$3500 - 3000 = 500$
The skydivers have fallen 500 m after 10 s.
b) Let $H = 1000$ and solve for t.
$H = 3500 - 5t^2$
$1000 = 3500 - 5t^2$
$1000 - 3500 = -5t^2$
$-2500 = -5t^2$
$500 = t^2$
$\sqrt{500} = t$
$22.4 \text{ s} \doteq t$
The skydivers have fallen for about 22.4 s.
14. The height of the arch is the maximum of the equation. Write the equation in vertex form to determine the coordinates of the maximum. Factor –0.008 out of the first two terms.
$h = -0.008x^2 - 1.296x + 107.5$
$h = -0.008(x^2 + 162x) + 107.5$

Now, add and subtract the square of half of the coefficient of the x term in the brackets, which is $\left(\frac{162}{2}\right)^2 = 6561$:

$$h = -0.008(x^2 + 162x + 6561 - 6561) + 107.5$$
$$h = -0.008[(x + 81)^2 - 6561] + 107.5$$
$$h = -0.008(x + 81)^2 + 52.488 + 107.5$$
$$h = -0.008(x + 81)^2 + 159.99$$

The vertex is (–81, 159.99). The height of the arch in the bridge is about 160 m.

15. When the ticket price is \$5, about 300 people will attend and the revenue will be $5 \times 300 = \$1500$. Let x represent the number of \$0.50 price increases and corresponding the number of times the attendance is reduced by 20 people. So, the ticket price will be $P = 5 + 0.5x$ and the attendance will be $A = 300 - 20x$. The equation for the revenue, R, will be $P \times A$ or:

$$R = (5 + 0.50x)(300 - 20x)$$

Expand and factor.

$$R = 1500 - 100x + 150x - 10x^2$$
$$R = 1500 + 50x - 10x^2$$

Let $R = 1562.50$ and solve for x.

$$1562.50 = 1500 + 50x - 10x^2$$
$$0 = -1562.50 + 1500 + 50x - 10x^2$$
$$0 = -62.50 + 50x - 10x^2$$

Use the quadratic formula to determine the values of x, where $a = -10$, $b = 50$, and $c = -62.50$.

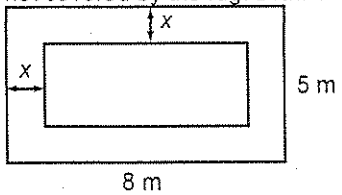
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-50 \pm \sqrt{50^2 - 4(-10)(-62.50)}}{2(-10)}$$
$$x = \frac{-50 \pm \sqrt{2500 - 2500}}{-20}$$
$$x = \frac{-50 \pm \sqrt{0}}{-20}$$
$$x = 2.5$$

Use x to calculate the ticket price.

$$P = 5 + 0.50x$$
$$P = 5 + 0.50(2.5)$$
$$P = 5 + 1.25$$
$$P = \$6.25$$

A price of \$6.25 will generate revenue of \$1562.50.

16. Let x represent the uniform width of the strip of floor not covered by the rug. Draw a diagram.



The area of the floor is:

$$A = (5)(8)$$
$$= 40 \text{ m}^2$$

The area of the rug is:

$$A = \frac{3}{4}(40)$$
$$= 30 \text{ m}^2$$

Each side of the rug is $2x$ m shorter than the floor. So, the formula for the area of the rug is:

$$A = (5 - 2x)(8 - 2x)$$

Set the formula equal to the area of the rug. Expand and combine like terms. Rearrange the terms so the left side equals zero.

$$30 = (5 - 2x)(8 - 2x)$$
$$30 = 40 - 10x - 16x + 4x^2$$
$$0 = 40 - 30 - 26x + 4x^2$$
$$0 = 10 - 26x + 4x^2$$

Use the quadratic formula to determine the values of x, where $a = 4$, $b = -26$, and $c = 10$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-26) \pm \sqrt{(-26)^2 - 4(4)(10)}}{2(4)}$$
$$x = \frac{26 \pm \sqrt{676 - 160}}{8}$$
$$x = \frac{26 \pm \sqrt{516}}{8}$$

So, x is about 0.41 m or 6.09 m.

$x = 6.09$ m makes no sense in this context because the floor would have a negative length and width:

$$8 - 2(6.09) = -4.18 \text{ m}.$$

Therefore, the width of the strip is about 0.4 m.

17. Let x and y represent the two integers. Write the relationship between the two integers as equations and isolate x.

$$x - y = 12 \qquad x^2 + y^2 = 1040$$
$$x = 12 + y \qquad x^2 = 1040 - y^2$$

Substitute the value of x into the second equation. Expand and combine like terms. Rearrange the terms so the right side is 0.

$$(12 + y)^2 = 1040 - y^2$$
$$144 + 12y + 12y + y^2 = 1040 - y^2$$
$$144 + 12y + 12y + y^2 - 1040 + y^2 = 0$$
$$2y^2 + 24y - 896 = 0$$

Factor out the 2.

$$2(y^2 + 12y - 448) = 0$$

Factor the trinomial.

$$2(y + 28)(y - 16) = 0$$

Solve for y.

$$y + 28 = 0 \qquad y - 16 = 0$$
$$y = -28 \qquad y = 16$$

Substitute the values of y to solve for x.

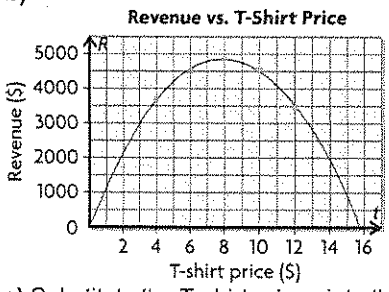
$$x = 12 + y \qquad x = 12 + y$$
$$x = 12 + (-28) \qquad x = 12 + (16)$$
$$x = -16 \qquad x = 28$$

So, the two integers are 16 and 28 or –16 and –28.

18. a) Revenue = price × number of students

T-Shirt Price, t ($)	Students Who Would Buy, N	Revenue, R
4.00	923	3692.00
6.00	752	4512.00
8.00	608	4864.00
10.00	455	4550.00
12.00	287	3444.00

b)



c) Substitute the T-shirt prices into the relation, and determine whether the values of N are close to those in the table.

$N = 1230 - 78(4)$
$= 918$

$N = 1230 - 78(6)$
$= 762$

$N = 1230 - 78(8)$
$= 606$

$N = 1230 - 78(10)$
$= 450$

$N = 1230 - 78(12)$
$= 294$

These values of N are close to those in the table, so this relation does approximate the number of students who will buy a T-shirt.

d) Revenue = $t \times N$

Substitute the value of N from the equation in part c) and expand.

$R = t(1230 - 78t)$
$R = 1230t - 78t^2$

e) Let $R = 4750$ and rearrange the terms so the right side equals 0.

$4750 = 1230t - 78t^2$
$4750 - 1230t + 78t^2 = 0$

Use the quadratic formula to determine the values of t, where $a = 78$, $b = -1230$, and $c = 4750$.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(-1230) \pm \sqrt{(-1230)^2 - 4(78)(4750)}}{2(78)}$$

$$t = \frac{1230 \pm \sqrt{1\,512\,900 - 1\,482\,000}}{156}$$

$$t = \frac{1230 \pm \sqrt{30\,900}}{156}$$

So, t is about \$6.76 or \$9.01.

All the prices from \$6.76 to \$9.01 will generate a revenue of \$4750 or greater. So, the student council should sell the T-shirts for between \$6.76 and \$9.01.

Chapter Self-Test, page 363

1. Answers may vary, e.g.,

a) Set $y = 0$ and look for the corresponding x-values from the graph. When x is about –2.9 or 0.9, it will satisfy the equation.

b) Set $y = -7$ and look for corresponding x-values from the graph. When x is –2 or 0, it will satisfy the equation.

c) Rewrite the equation as $3x^2 + 6x - 7 - 2 = 0$.

Rearranging the equation, we get $3x^2 + 6x - 7 = 2$.

Set $y = 2$ and look for corresponding x-values from the graph. When x is –3 or 1, it will satisfy the equation.

2. a) Factor the trinomial:

$x^2 + 5x - 14 = 0$
$(x + 7)(x - 2) = 0$

The roots are –7 and 2.

b) Use the quadratic formula: $a = 5$, $b = -9$, $c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(5)(1)}}{2(5)}$$

$$x = \frac{9 \pm \sqrt{81 - 20}}{10}$$

$$x = \frac{9 \pm \sqrt{61}}{10}$$

The roots are about 0.12 and 1.68.

c) Rewrite the equation such that one side is a zero.

$2x^2 - 8 - 24 = 0$
$2x^2 - 32 = 0$
$x^2 - 16 = 0$

Factor the difference of squares:

$(x - 4)(x + 4) = 0$

Thus, the roots are –4 and 4.

d) Isolate x:

$$2(x-1)^2 = 5$$

$$(x-1)^2 = \frac{5}{2}$$

$$(x-1) = \pm\sqrt{\frac{5}{2}}$$

$$x = \pm\sqrt{\frac{5}{2}} + 1$$

Thus, the roots are about 2.58 and about –0.58.

3. a) $y = 2(x^2 + 6x) - 14$
$y = 2(x^2 + 6x + 9 - 9) - 14$
$y = 2[(x + 3)^2 - 9] - 14$
$y = 2(x + 3)^2 - 32$

Thus, the vertex of the parabola is (–3, –32).

b) $y = 3(x^2 - 5x) - 24$
$y = 3(x - 5x + 6.25 - 6.25) - 24$
$y = 3[(x - 2.5)^2 - 6.25] - 24$
$y = 3(x - 2.5)^2 - 42.75$

Thus, the vertex is (2.5, –42.75).

4. Yes, because all quadratic equations have a vertex, so it is possible to write an equation in vertex form by completing the square.

5. a) $D = b^2 - 4ac$
$D = (-4)^2 - 4(2)(7)$
$D = 16 - 56$
$D = -40$
Since $D < 0$, there will be no real roots.
b) Since both factors are the same, the roots will be the same. Thus there will be only one real root.
c) Since both factors are the same, the roots will be the same. Thus, there will be only one real root.
6. a) The maximum profit she can earn in a week is the P-value at the vertex. Completing the square, we can solve for the vertex.
$P = -0.1(n^2 - 300n + 12\,000)$
$P = -0.1(n^2 - 300n + 22\,500 - 10\,500)$
$P = -0.1[(n - 150)^2 - 10\,500]$
$P = -0.1(n - 150)^2 + 1050$
The vertex is at (150, 1050). Thus, the maximum profit she can earn in a week is \$1050, so she cannot earn a profit of \$2000 in a week.
b) She breaks even when $P = 0$.
$0 = -0.1n^2 + 30n - 1200$
Using the quadratic formula, we have $a = -0.1$, $b = 30$, and $c = -1200$.

$$n = \frac{-30 \pm \sqrt{30^2 - 4(-0.1)(-1200)}}{2(-0.1)}$$

$$n = \frac{-30 \pm \sqrt{900 - 480}}{-0.2}$$

$$n = \frac{-30 \pm \sqrt{420}}{-0.2}$$

The roots are about 48 and 252. So she must sell either 48 bears or 252 bears to break even.
c) Set $P = 500$.
$500 = -0.1n^2 + 30n - 1200$
$0 = -0.1n^2 + 30n - 1700$
Using the quadratic formula, we have $a = -0.1$, $b = 30$, and $c = -1700$.

$$n = \frac{-30 \pm \sqrt{30^2 - 4(-0.1)(-1700)}}{2(-0.1)}$$

$$n = \frac{-30 \pm \sqrt{900 - 680}}{-0.2}$$

$$n = \frac{-30 \pm \sqrt{220}}{-0.2}$$

The roots are about 76 or 224, thus she must sell about 76 bears or 224 bears to make \$500.
d) The number of bears she needs to sell to maximize profit is the n-value in the vertex. Using the vertex we found in a), she must sell 150 bears to maximize her profit.
7. Let x and y be the lengths of the rectangle. Since the total amount of fencing must be 24 m, $2x + y = 24$. The area is also the length of one side multiplied by the other, that is, $A = xy$. From the first equation, $y = 24 - 2x$. Substituting this value into our area equation, we get, $A = x(24 - 2x)$
Thus the roots are 0 and 12. The x-coordinate of the vertex must be in the middle of the two roots, thus the x-coordinate of the vertex is $(0 + 12) \div 2 = 6$.
Substituting the value of x into our first equation, we get $y = 12$. Thus the dimensions are 6 m by 12 m.
8. Answers may vary, e.g.,
Reason 1: I could not make a square using algebra tiles.
Reason 2: When 3 is factored out of all terms, the coefficient of x is 2. This means that the constant term would have to be $\left(\frac{2}{2}\right)^2 \times 3 = 3$, not 6, to be a perfect square.
9. Let x represent the number of \$0.50 increases, and R represent the revenue. The fare will be the initial \$2.25 in addition to $0.5x$. The number of passengers will be the initial 5000 passengers subtracting $150x$. Since the revenue is the fare times the number of passengers, $R = (2.25 + 0.5x)(5000 - 150x)$. Thus, we must solve for x when $R = 15\,275$.
$15\,275 = (2.25 + 0.5x)(5000 - 150x)$
$15\,275 = 11\,250 - 337.5x + 2500x - 75x^2$
Set the left side equal to zero.
$0 = -75x^2 + 2162.5x - 4025$
Solve for x using the quadratic formula,

$$x = \frac{-2162.5 \pm \sqrt{2162.5^2 - 4(-75)(-4025)}}{2(-75)}$$

$$x = \frac{-2162.5 \pm \sqrt{4\,676\,406.25 - 1207\,500}}{-150}$$

$$x = \frac{-2162.5 \pm \sqrt{3\,468\,906.25}}{-150}$$

Thus, x is 2 or about 26.83.
To stay in business, they must charge either \$3.25 or \$15.67. (However, increasing the price by \$13.42 is unreasonable because it is almost six times the original price.)

Chapter Task, page 364

A. In this situation, $u = 0$ and $h_0 = 6$ so the relation that models the height of the ball is $h = -5t^2 + 6$.
B. Solve for the roots of the quadratic relation.
Set $h = 0$,

$$0 = -5t^2 + 6$$
$$-6 = -5t^2$$
$$\frac{6}{5} = t^2$$
$$\pm\sqrt{\frac{6}{5}} = t$$

The time at which the height is at maximum lies on the vertex, which is midway between the roots. Thus,

$$t = \frac{\left[\sqrt{\frac{6}{5}} + \left(-\sqrt{\frac{6}{5}}\right)\right]}{2}$$

$$t = 0$$

So the maximum height occurs at $t = 0$. Substituting this into our quadratic, we get $h = 6$. This makes sense because without any initial velocity, Myrtle is just dropping the ball, thus the maximum height should be at the point of release.

C. $h = -5t^2 + 2t + 6$

$$h = -5\left(t^2 - \frac{2}{5}t - \frac{6}{5}\right)$$

$$h = -5\left(t^2 - \frac{2}{5}t + \frac{1}{25} - \frac{31}{25}\right)$$

$$h = -5\left[\left(t - \frac{1}{5}\right)^2 - \frac{31}{25}\right]$$

$$h = -5\left(t - \frac{1}{5}\right)^2 + \frac{31}{5}$$

So the maximum height is $\frac{31}{5}$ m or 6.2 m.

D. Using logic, we can determine that when the initial velocity was zero, the maximum height was lower than when the initial velocity was 2, and it seems that by increasing the initial velocity, the maximum height will keep increasing.

E. For an initial velocity of 4 m/s, we have $u = 4$ and $h_0 = 6$ so the relation that models the height of the ball is $h = -5t^2 + 4t + 6$. Completing the square we have,

$$h = -5t^2 + 4t + 6$$

$$h = -5\left(t^2 - \frac{4}{5}t - \frac{6}{5}\right)$$

$$h = -5\left(t^2 - \frac{4}{5}t + \frac{4}{25} - \frac{34}{25}\right)$$

$$h = -5\left[\left(t - \frac{2}{5}\right)^2 - \frac{34}{25}\right]$$

$$h = -5\left(t - \frac{2}{5}\right)^2 + \frac{34}{5}$$

So the maximum height is $\frac{34}{5}$ m or 6.8 m.

For an initial velocity of 6 m/s, we have $u = 6$ and $h_0 = 6$ so the relation that models the height of the ball is $h = -5t^2 + 6t + 6$. Completing the square we have,

$$h = -5t^2 + 6t + 6$$

$$h = -5\left(t^2 - \frac{6}{5}t - \frac{6}{5}\right)$$

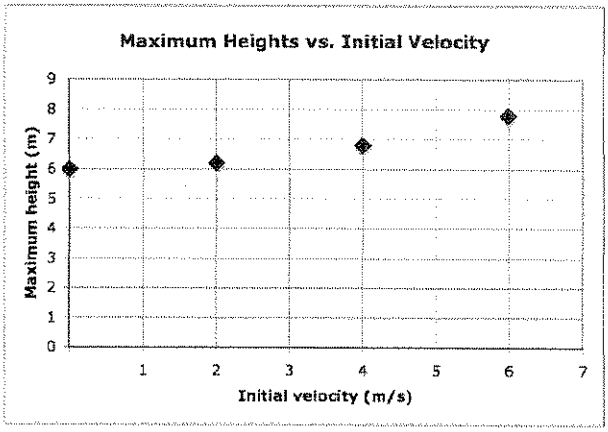
$$h = -5\left(t^2 - \frac{6}{5}t + \frac{9}{25} - \frac{39}{25}\right)$$

$$h = -5\left[\left(t - \frac{3}{5}\right)^2 - \frac{39}{25}\right]$$

$$h = -5\left(t - \frac{3}{5}\right)^2 + \frac{39}{5}$$

So the maximum height is $\frac{39}{5}$ m or 7.8 m.

F.



From the graph, it looks like a quadratic relation. We can confirm this by analyzing our data and noting that we have constant second differences (0.4).

G. Using graphing technology, an algebraic model to represent the maximum heights is $h_{max} = 0.05u^2 + 6$.

H. Setting $h_{max} = 11$, we can solve for u.

$$11 = 0.05u^2 + 6$$
$$5 = 0.05u^2$$
$$100 = u^2$$
$$u = \pm 10$$

Ignore the negative initial velocity. Thus Myrtle must throw the ball with an initial velocity of 10 m/s if she wishes to clear the tree.

Cumulative Review, pages 365–367

1. Expand each bracket.

A. $2(6x^2y^3 + 3xy - y^2)$
$= 2(6x^2y^3) + 2(3xy) + 2(-y^2)$
$= 12x^2y^3 + 6xy - 2y^2$

B. $2y(6x^2y^2 + 3x - y)$
$= 2y(6x^2y^2) + 2y(3x) + 2y(-y)$
$= 12x^2y^3 + 6xy - 2y^2$

C. $-2y(-6x^2y^2 - 3x + y)$
$= -2y(-6x^2y^2) - 2y(-3x) - 2y(y)$
$= 12x^2y^3 + 6xy - 2y^2$

Thus, they are all the factored form of $12x^2y^3 + 6xy - 2y^2$, thus D is the correct answer.

2. To factor, we need two numbers that sum to 6 and have a product of –27. Two numbers that do that are 9 and –3. Thus the factored form is $(t + 9)(t - 3)$, so B is the correct answer.

3. Possible pairs of factors of 6 are 1 and 6, or 2 and 3. Possible factors of –5 are –5 and 1, or –1 and 5. Since $5 \times 3 + (-1 \times 2) = 15 - 2 = 13$, the factored form of $6a^2 + 13a - 5$ is $(2a + 5)(3a - 1)$, so A is the correct answer.

4. This is a perfect-square trinomial since the coefficient of c^2 and the constant term are both perfect squares, and the coefficient of the c term is $2\sqrt{9}\sqrt{25}$. The coefficient of the c term is positive, so the factored form is $(3c + 5)^2$. So C is the correct answer.

5. This is a difference of squares, so the factored form is $(6m^3 - 7)(6m^3 + 7)$, so A is the correct answer.

6. Grouping the first two terms and the last two terms together, we can common factor.
$2xy - 10x + 3y - 15 = 2x(y - 5) + 3(y - 5)$
Now common factor the $(y - 5)$ term.
$(y - 5)(2x + 3)$
So D is the correct answer.
7. Start with $y = x^2$. The black graph is a reflection about the x-axis of the green graph, thus we need a minus sign. The graph is stretched horizontally, so the magnitude of the coefficient must be less than 1. So C is the correct answer.
8. This equation is in vertex form, so the h value is -1 and the k value is -4, so the vertex is $(-1, -4)$. The equation of the line of symmetry is the x-value, so it is $x = -1$, so B is the correct answer.
9. To reflect the equation in the x-axis, we must put a negative sign on x^2. To stretch the graph by a factor of 2, we must multiply by 2. To shift 3 units left, we must replace our x with $(x + 3)$. So our final equation looks like $y = -2(x + 3)^2$, so D is the correct answer.
10. The vertex form of a parabola is $y = a(x - h)^2 + k$. We know the vertex $(h, k) = (2, 5)$. So, $y = a(x - 2)^2 + 5$. Solve for a by substituting the point $(4, -1)$ into our equation.
$-1 = a(4 - 2)^2 + 5$
$-6 = a2^2$
$-6 = 4a$
$a = -1.5$
Thus the equation of the parabola is $y = -1.5(x - 2)^2 + 5$. So, the correct answer is D.
11. If we start with $y = x^2$, the negative sign would indicate a reflection in the x-axis, so B is a correct transformation, the 2 would indicate a vertical stretch by a factor of 2, so A is a correct transformation, the + 4 would indicate a vertical shift 4 units up, so D is a correct transformation, and the -3 would indicate a horizontal shift 3 units right, but C says left, so C is the incorrect transformation and is our answer.
12. The revenue model is written in vertex form. To increase the maximum daily revenue by \$250, we must add 250 to the k value. To increase the price of the dress by \$28, we must add 28 to our h value. Thus the new revenue model will be,
$R = -2[d - (135 + 28)]^2 + (1500 + 250)$
$R = -2(d - 163)^2 + 1750$
So D is the correct answer.
13. Expand the quadratic relation into standard form,
$y = 2x(x - 6) - 5$
$y = 2x^2 - 12x - 5$
Rewrite this equation into vertex form by completing the square.
$y = 2(x^2 - 6x - 2.5)$
$y = 2(x^2 - 6x + 9 - 11.5)$
$y = 2(x^2 - 6x + 9)^2 - 23$
$y = 2(x - 3)^2 - 23$
So A is the correct answer.
14. Solve for the zeros by setting $h = 0$.
$0 = -5t^2 + 80t$
$0 = -5t(t - 16)$
Thus, the roots are 0 and 16. The vertex is located midway between the roots, so object is at a maximum height when $t = 8$ s. Substituting this value into our height equation, we get,
$h = -58^2 + 80(8)$
$h = 320$ m
The maximum height is 320 m, so, C is the correct answer.
15. Factor the standard form of set 1.
$y = -2x^2 + 8x + 10$
$y = -2(x^2 - 4x - 5)$
$y = -2(x + 1)(x - 5)$
Write the standard form of set 1 in vertex form.
$y = -2x^2 + 8x + 10$
$y = -2(x^2 - 4x - 5)$
$y = -2(x^2 - 4x + 4 - 9)$
$y = -2[(x - 2)^2 - 9]$
$y = -2(x - 2)^2 + 18$
Thus, set 1 is expressed correctly in all three ways.
Factor the standard form of set 2.
$y = 3x^2 + 9x - 84$
$y = 3(x^2 + 3x - 28)$
$y = 3(x + 7)(x - 4)$
Write the standard form of set 2 in vertex form.
$y = 3x^2 + 9x - 84$
$y = 3(x^2 + 3x - 28)$
$y = 3(x^2 + 3x + 2.25 - 30.25)$
$y = 3[(x + 1.5)^2 - 30.25]$
$y = 3(x + 1.5)^2 - 90.75$
This is not the same as the vertex form in set 2, so only set 1 is correctly expressed in all three ways, so A is the correct answer.
16. To factor the equation, we need to determine two numbers that sum to 1 and have a product of -6. Two numbers that do that are -2 and 3. So, $(x - 2)(x + 3) = 0$. Thus the roots are the negative factors, that is, $x = 2$ and $x = -3$, so D is the correct answer.
17. One condition to be a perfect-square trinomial is that the coefficient of the squared term as well as the constant term must perfect squares. But in C, the constant term is -25 which is not a perfect square, so C is the correct answer.
18. To be a perfect square, $c = \frac{1}{a}\left(\frac{b}{2}\right)^2$. So, with $a = 1$ and $b = -8$, $c = 1\left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$, so A is the correct answer.
19. Write the equation in vertex form.
$y = x^2 + 6x + 7$
$y = x^2 + 6x + 9 - 2$
$y = (x + 3)^2 - 2$
So C is the correct answer.

20. This equation cannot be factored, so use the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-2)}}{2(1)}$$
$$x = \frac{-3 \pm \sqrt{9 + 8}}{2}$$
$$x = \frac{-3 \pm \sqrt{17}}{2}$$

Thus, the approximate solutions are $x = 0.56$ and $x = -3.56$, so A is the correct answer.

21. Check the discriminant for each equation. The equation will have exactly one real root if the discriminant is 0. Use the formula $D = b^2 - 4ac$

A. $D = (-6)^2 - 4(-9)(1)$
$D = 36 + 36$
$D = 72 \neq 0$
Thus A will not have exactly one real root.

B. $D = (-6)^2 - 4(9)(-1)$
$D = 36 + 36$
$D = 72 \neq 0$
Thus, B will not have exactly one real root.

C. $D = (-6)^2 - 4(9)(1)$
$D = 36 - 36$
$D = 0$
Thus, C will have exactly one real root.

D. $D = 6^2 - 4(9)(-1)$
$D = 36 + 36$
$D = 72 \neq 0$
Thus, D will not have exactly one real root.
So C is the correct answer.

22. Check the discriminant for each equation. The equation will no real solutions if the discriminant is less than 0. Use the formula $D = b^2 - 4ac$

A. $D = 5^2 - 4(2)(-8)$
$D = 25 + 64$
$D = 89 > 0$
Thus A will have two real roots.

B. $D = 2^2 - 4(-3)(-5)$
$D = 4 - 60$
$D = -54 < 0$
Thus, B will not have any real roots.

C. $D = 2^2 - 4(-1)(5)$
$D = 4 + 20$
$D = 24 > 0$
Thus, C will have two real roots.

D. $D = 8^2 - 4(2)(3)$
$D = 64 - 24$
$D = 40 > 0$
Thus, D will have two real roots.
So B is the correct answer.

23. Substitute the k-values into the discriminant to check which give a value greater than 0. Use the formula $D = b^2 - 4ac$

A. $D = 1^2 - 4(2)(1)$
$D = 1 - 8$
$D = -7 < 0$
Thus, A will not have two real solutions.

B. $D = (-4)^2 - 4(2)(1)$
$D = 16 - 8$
$D = 8 > 0$
Thus, B will have two real solutions.

C. $D = 2^2 - 4(2)(1)$
$D = 4 - 8$
$D = -4 < 0$
Thus, C will not have two real solutions.

D. $D = (-2)^2 - 4(2)(1)$
$D = 4 - 8$
$D = -4 < 0$
Thus, D will not have two real solutions.
So B is the correct answer.

24. To determine how long the rocket was in the air, you must determine at what time the rocket hits the ground. This occurs when $h = 0$, which is a zero of the relation.
So D is the correct answer.

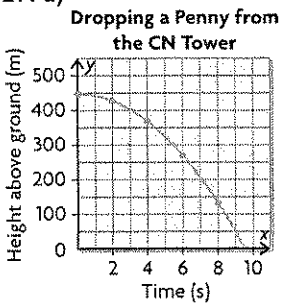
25. A. In vertex form, the axis of symmetry is the h-value, which is 3. So A has $x = 3$ as an axis of symmetry.

B. In factored form, the axis of symmetry is located midway between the two roots. The roots are $x = 5$ and $x = 1$. Since $x = 3$ is midway between these two roots, this is the axis of symmetry.

C. The axis of symmetry can be found using the formula $x = \frac{-b}{2a} = \frac{-(-6)}{2(1)} = \frac{6}{2} = 3$. The equation of the axis of symmetry is $x = 3$.
Since all the relations have $x = 3$ as the axis of symmetry, D is the correct answer.

26. Factor each equation.
For Sid's company,
$P = -6(n - 4)(n - 8)$
Thus the roots occur at $n = 4$ and $n = 8$. The vertex is midway between these two roots, so the vertex occurs at $n = 6$. Substituting $n = 6$ into our profit equation,
$P = -6(6 - 2)(6 - 8)$
$P = -6(2)(-2)$
$P = 24$
Thus the vertex of the relation is (6, 24). From the vertex, the maximum profit is \$24 000 when 6000 pairs of shoes are manufactured and sold. The break-even points occur at the roots, so they occur when 4000 and 8000 pairs of shoes have been manufactured and sold.
For Nancy's company,
$P = -8(n - 1)(n - 4)$
Thus the roots occur at $n = 1$ and $n = 4$. The vertex is midway between these two roots, so the vertex occurs at $n = 2.5$. Substituting $n = 2.5$ into our profit equation,
$P = -8(2.5 - 1)(2.5 - 4)$
$P = -8(1.5)(-1.5)$
$P = 18$
Thus the vertex of the relation is (2.5, 18). From the vertex, the maximum profit is \$18 000 when 2500 pairs of shoes are manufactured and sold. The break-even points occur at the roots, so they occur when 1000 and 4000 pairs of shoes have been manufactured and sold.

27. a)



b) Yes, the second differences are constant.
c) Answers may vary, e.g., The vertex is at (0, 447.0).
Substituting this into vertex form, we have,
$y = a(x - 0)^2 + 447$
$y = ax^2 + 447$
Substitute the point (2, 427.4) into the equation.
$427.4 = a2^2 + 447$
$-19.6 = 4a$
$a = -4.9$
The equation for the curve of good fit is $y = -4.9x^2 + 447$.
d) Using graphing technology, the equation for the curve of good fit is $y = -4.9x^2 + 447$. Comparing with my answer from c), the fit is perfect.
e) Substitute 5.5 s into the equation
$y = -4.9(5.5)^2 + 447$
$y = -148.225 + 447$
$y = 298.775$
The penny is about 299 m above the ground after 5.5 s.
28. a) For equation (1),
$P = -10(x^2 - 12x - 160)$
$P = -10(x - 20)(x + 8)$
The vertex occurs midway between the two roots, thus the vertex occurs when $x = 6$. Substituting this into the profit equation,
$P = -10(6 - 20)(6 + 8)$
$P = 1960$
The maximum profit for equation (1) is \$1960 when $x = 6$, so the selling price is \$25.99.
For equation (2)
Complete the square.

$$P = -11\left(x^2 - \frac{49.5}{11}x - \frac{1598}{11}\right)$$

$$P = -11\left(x^2 - \frac{49.5}{11}x + \frac{612.5625}{121} - \frac{612.5625}{121} - \frac{1598}{11}\right)$$

$$P = -11\left(x^2 - \frac{49.5}{11}x + \frac{612.5625}{121} - \frac{612.5625}{121} - \frac{17578}{121}\right)$$

$$P = -11\left[\left(x - \frac{24.75}{11}\right)^2 - \frac{18\,190.5625}{121}\right]$$

$$P = -11(x - 2.25)^2 + 1653.69$$

The maximum profit for equation (2) is \$1653.69 when $x = 2.25$, so the selling price is \$22.24.
b) The break-even prices occur at the roots. For equation (1), we have already solved for the roots in part a), so the break-even points are $x = -8$ and $x = 20$, so the break-even prices for equation (1) are \$11.99 and \$39.99.
For equation (2), we must use the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-49.5 \pm \sqrt{(49.5)^2 - 4(-11)(1598)}}{2(-11)}$$

$$x = \frac{-49.5 \pm \sqrt{2450.25 + 70312}}{-22}$$

$$x = \frac{-49.5 \pm \sqrt{72762.25}}{-22}$$

The roots are about $x = -10.01$ and $x = 14.51$. Thus, the break-even prices for equation (2) are \$9.98 and \$34.50.
c) Answers may vary, e.g., the recommended selling price is \$25.99, based on equation (1). This gives the greatest possible profit.